# Development of the Strut-and-Tie Method for Appendix A of the Building Code (ACI 318-02)

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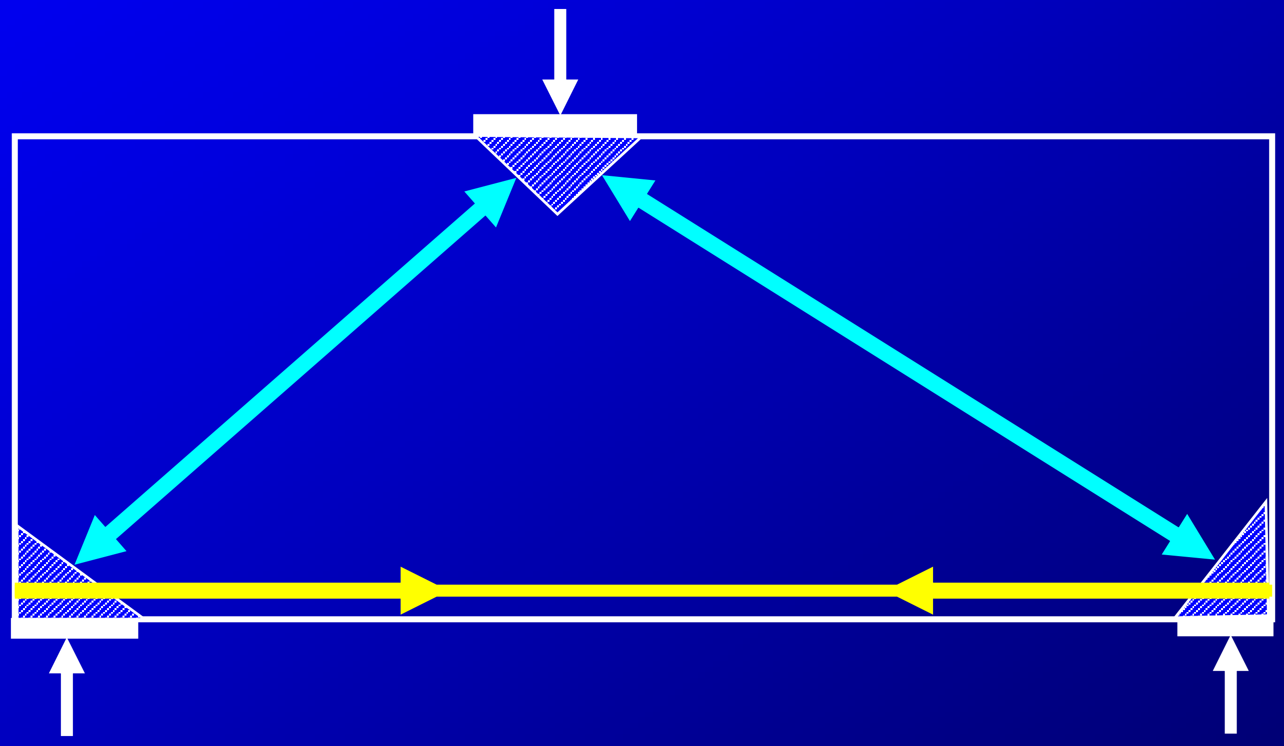
# Strut and Tie Modeling

Members or regions of members may be designed by idealizing the concrete and reinforcement as an assembly of axially loaded members, inter-connected at nodes, to form a truss capable of carrying loads across a region or member.

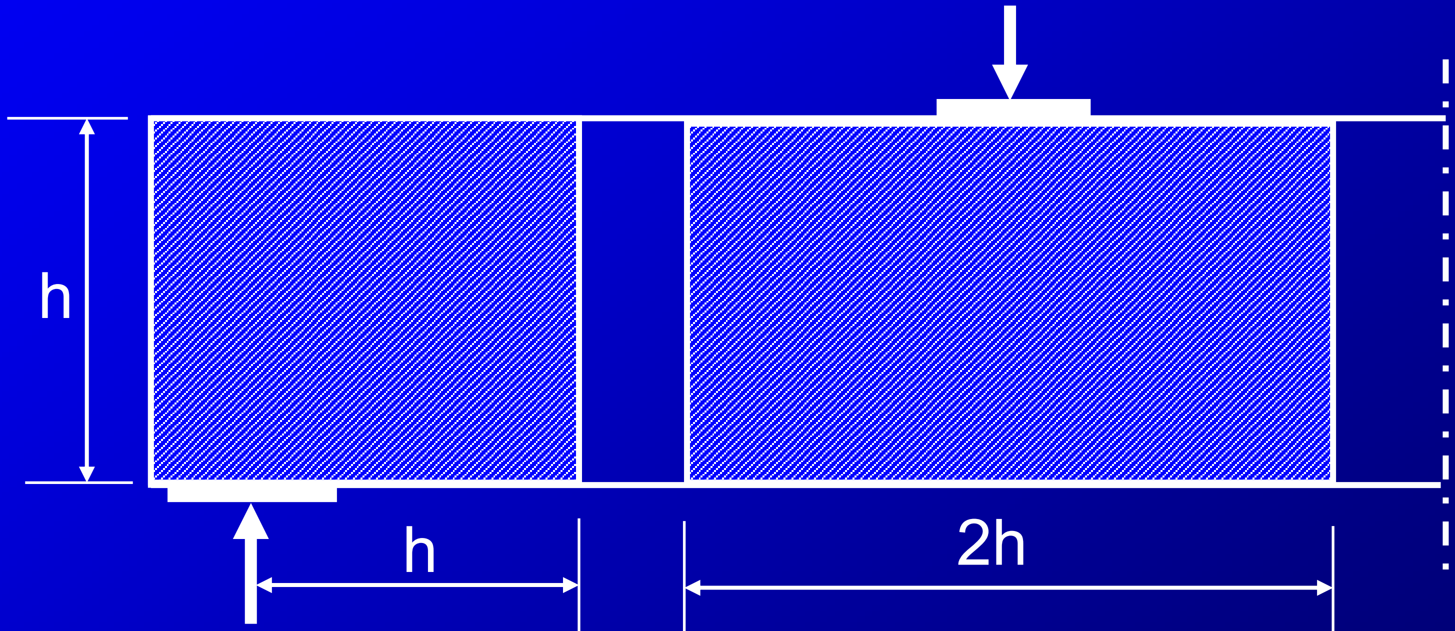
# Components of Strut and Tie Models



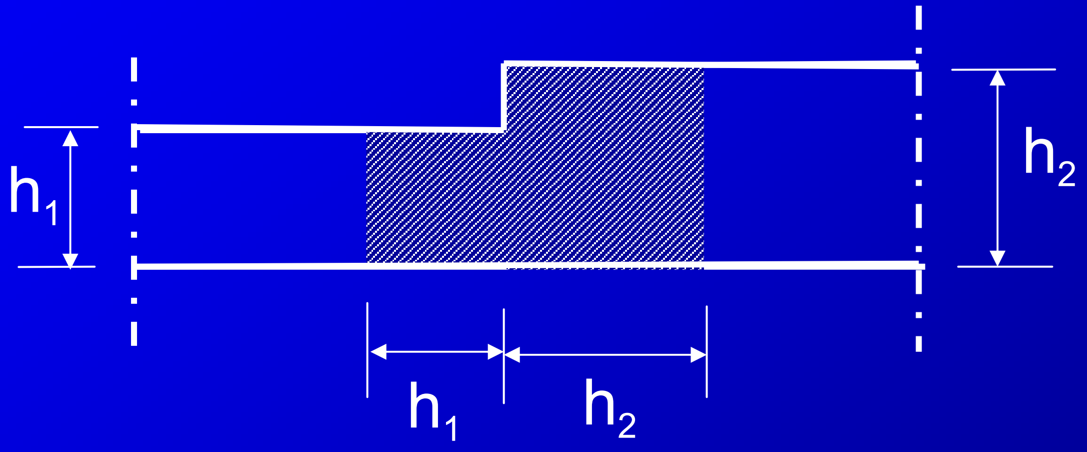
# Steps to Build Strut & Tie Model

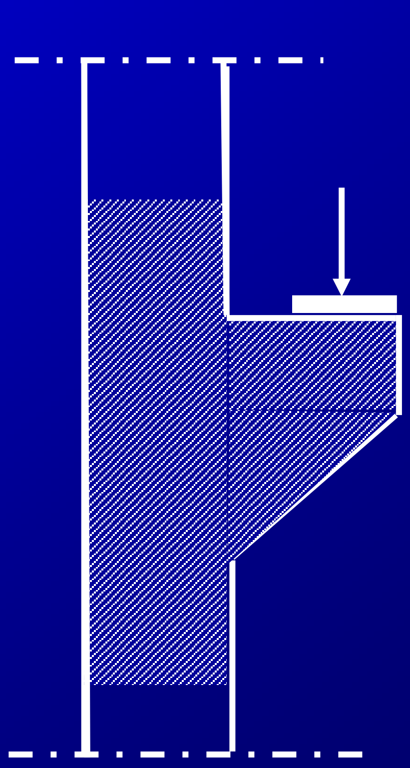
- Isolate member or D(disturbed) - region
- Compute forces or distribution of stresses on boundary
- Represent stress distributions as forces
- Select a truss model to transmit these forces across the member or D-region

# Concept of D-Regions
# (force discontinuities)



# Concept of D-Regions (geometric discontinuities)

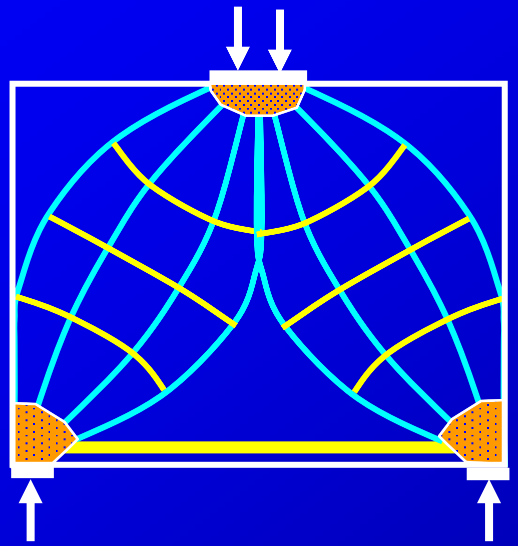


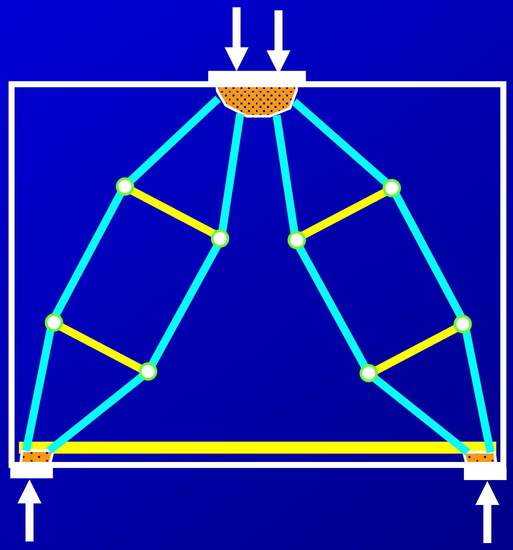



# Basic Requirements

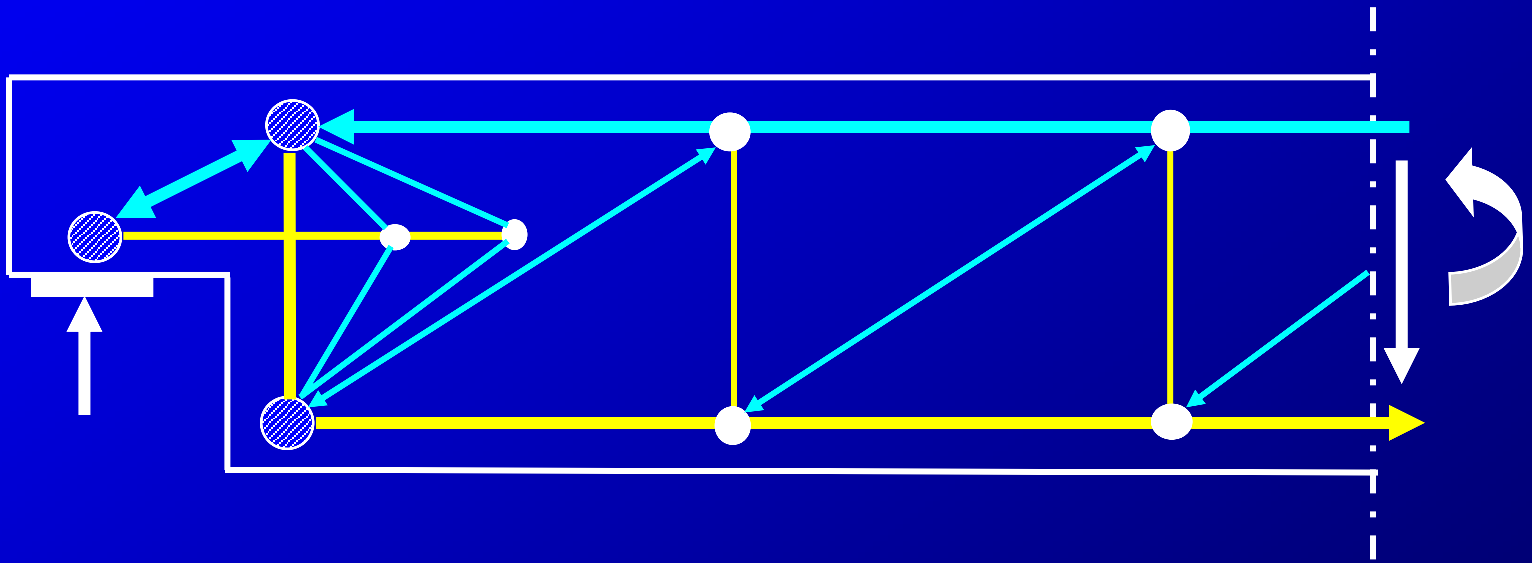
- Model approximates stress flow
- Define component dimensions and strengths
- Define $\phi$ and $\beta$ factors
- Analyze nodes and anchorage
- Select reinforcement details

# Modeling Stress Flow





# Modeling Stress Flow in D-Regions (Dapped Beam)



# How to Select the "Correct" Strut-and-Tie Model

- Some researchers suggest using a finite element model to determine stress trajectories, then selecting a STM to "model" the stress flow.
- Generally, a STM that minimizes the required amount or reinforcement is close to an ideal model.

# Required Definitions for Code

- Geometric rules to follow when creating a strut-and-tie model.
- Component strengths for determining members sizes and final geometry of the model.

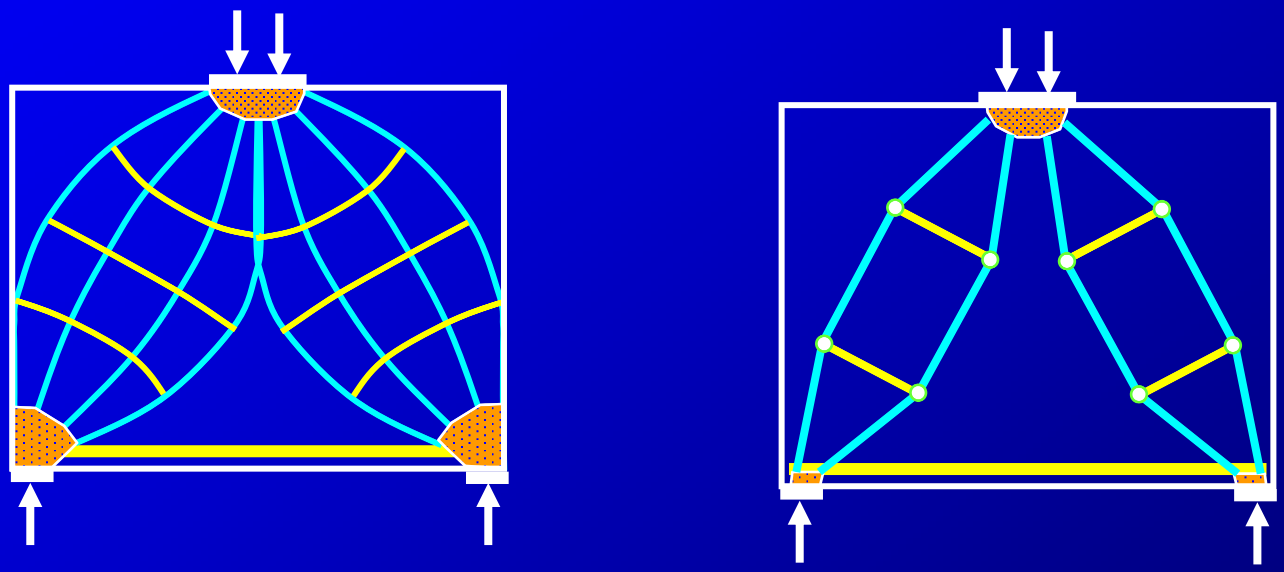
# Codes with Rules for Use of Strut-and-Tie Models

- AASHTO LRFD Specification
- Canadian Code for Design of Concrete Structures
- FIP Recommendations (1996) for Practical Design of Structural Concrete

# Strength of Compression Struts (what to consider)

- Longitudinal cracking due to transverse tension strain
- Transverse tension forces
- Sustained loads
- Reinforcement grid crossing strut
- Confinement by concrete or steel

# Lateral Expansion of Strut
# (bottle-shaped strut)



# Effective Compressive Strength of Struts

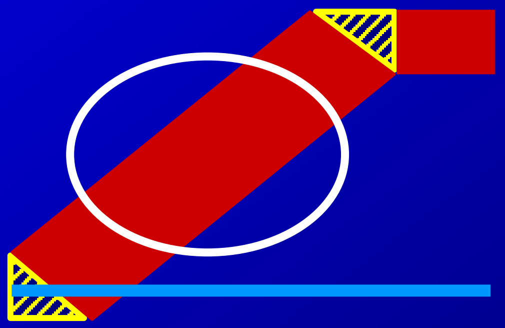
$$\phi F_{ns} \geq F_u$$

$$\phi = 0.75$$

$$F_{ns} = f_{cu} A_c$$

$$A_c = min.\ cross\text{-}sectional\ \ area\ \ of\ \ strut$$

$$\mathrm{f_{cu}} = 0.85\, \beta_s\, \mathrm{f_c'}$$



# AASHTO and CSA Evaluation of Effective Concrete Strength

$$\beta_s f_c' = \frac{f_c'}{0.8 + 170\varepsilon_1} \leq 0.85 f_c'$$

Transverse tension, $\varepsilon_1 = (\varepsilon_s + 0.002) \cot^2\theta_s$

$\theta_s$ = angle between strut and tie (steel)

$\varepsilon_s$ = strain in steel tie

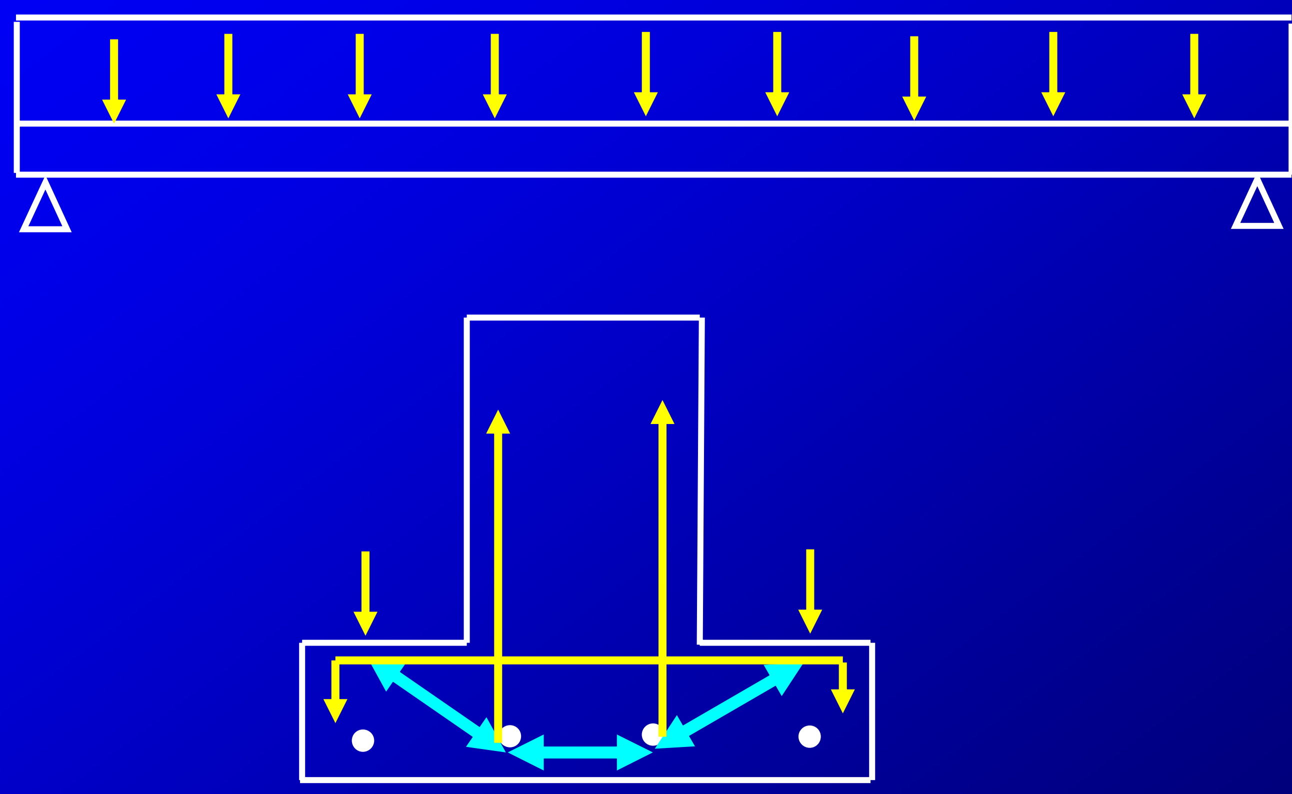
# FIP Recommendations for $\beta_s$ in Cracked Struts

- 0.80 - struts with longitudinal cracking (splitting), but crossed by minimum reinforcement grid
- 0.75 - struts crossed by “normal” width cracks
- 0.60 - struts crossed by “wide” cracks

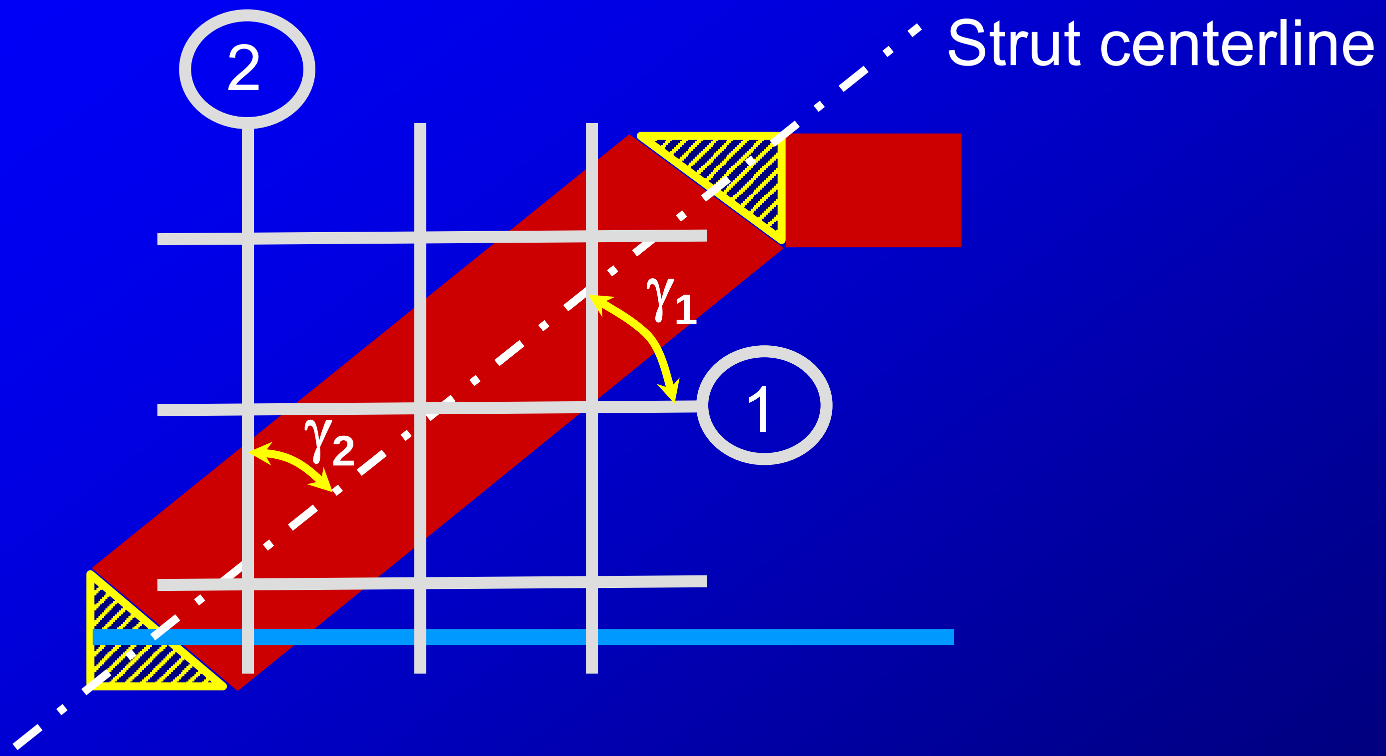
# ACI Recommended $\beta_s$ Values for Struts

- 1.0 – prismatic shape (constant width) over its length, similar to a flexural compression zone in a B-region
- 0.75 – inclined (bottle-shaped) strut crossed by minimum reinforcement grid
- 0.60$\lambda$ – inclined bottle-shaped strut not crossed by minimum reinforcement grid; where $\lambda$ accounts for lightweight concrete
- 0.40 – struts in flexural tension zones

# Struts in a Flexural Tension Zone

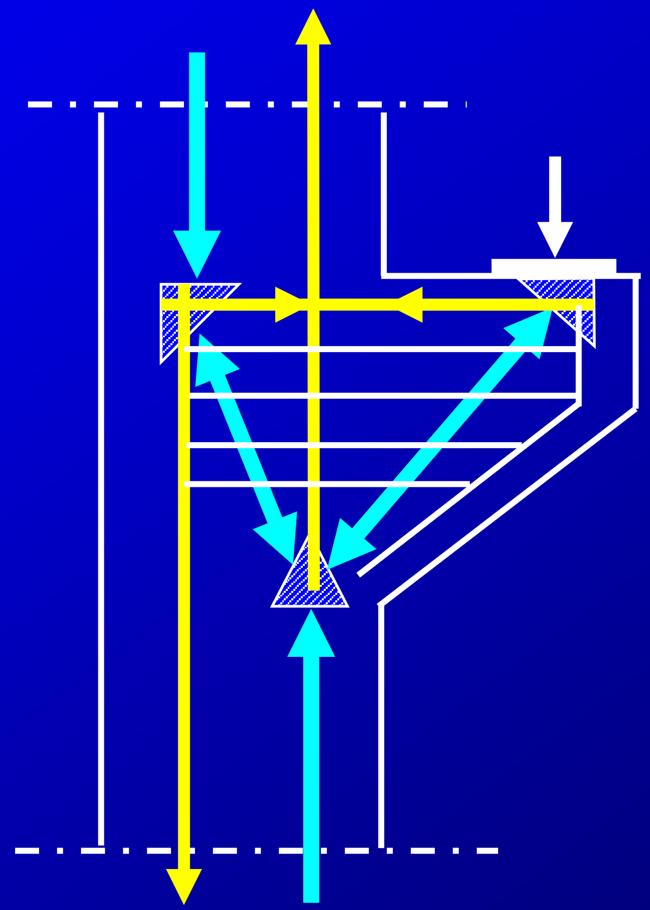


# Minimum Reinforcement Grid ($f'_c \leq 6000$ psi)




$$\Sigma \frac{A_{si}}{b\, s_i} \sin \gamma_i \geq 0.003$$

# Reinforcement "Grid" (only horizontal bars)



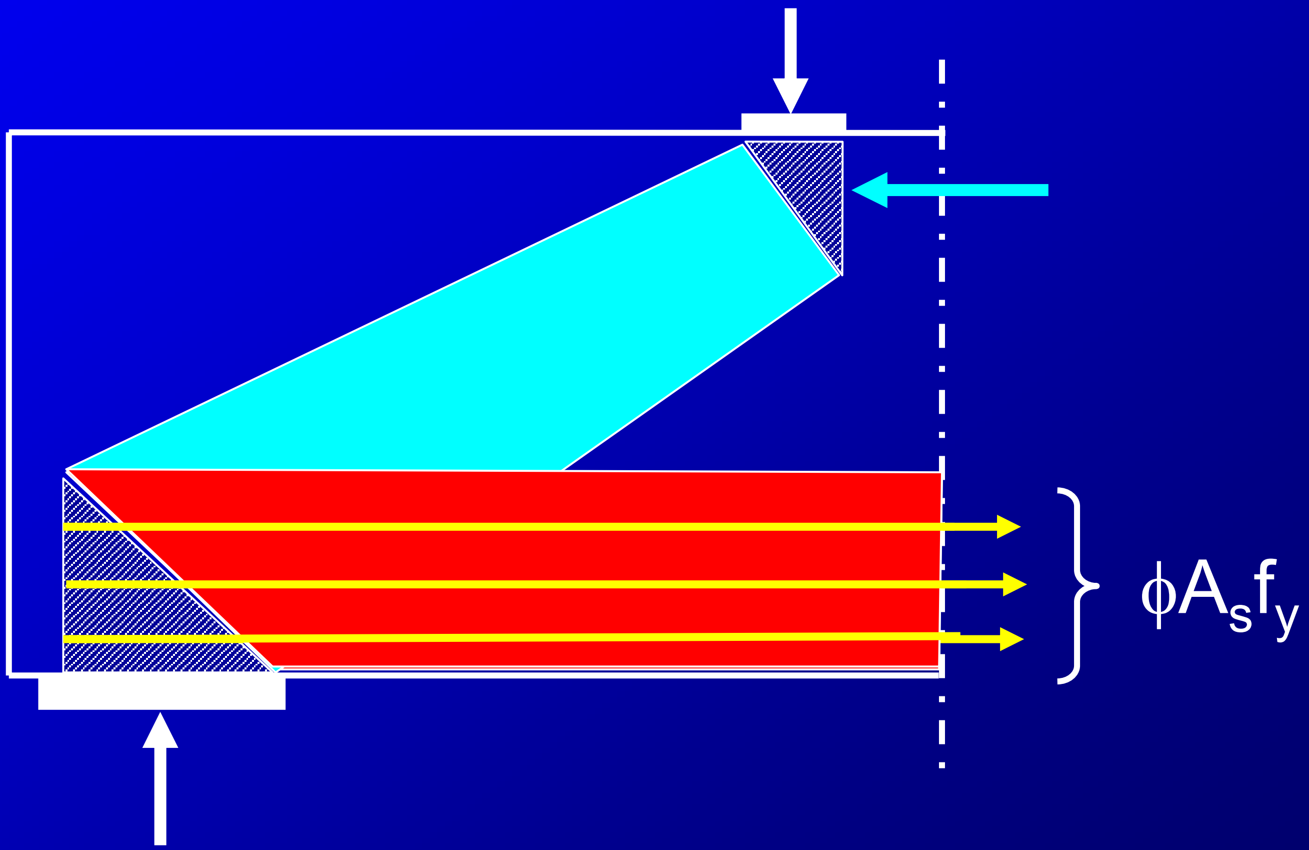
# Min. Reinf. Grid in other Codes

- AASHTO $\rho \geq 0.003$ EW
- CSA $\rho \geq 0.002$ EW
- FIP $\rho \geq 0.001$ EF, EW

# Tie Dimensions

- Full width (out or plane) of member
- Width (in plane) of tie is function of effective compression strength of concrete in nodes where tie is anchored
- Spread reinforcement throughout tie dimensions

# Tie - Dimensions and Strength

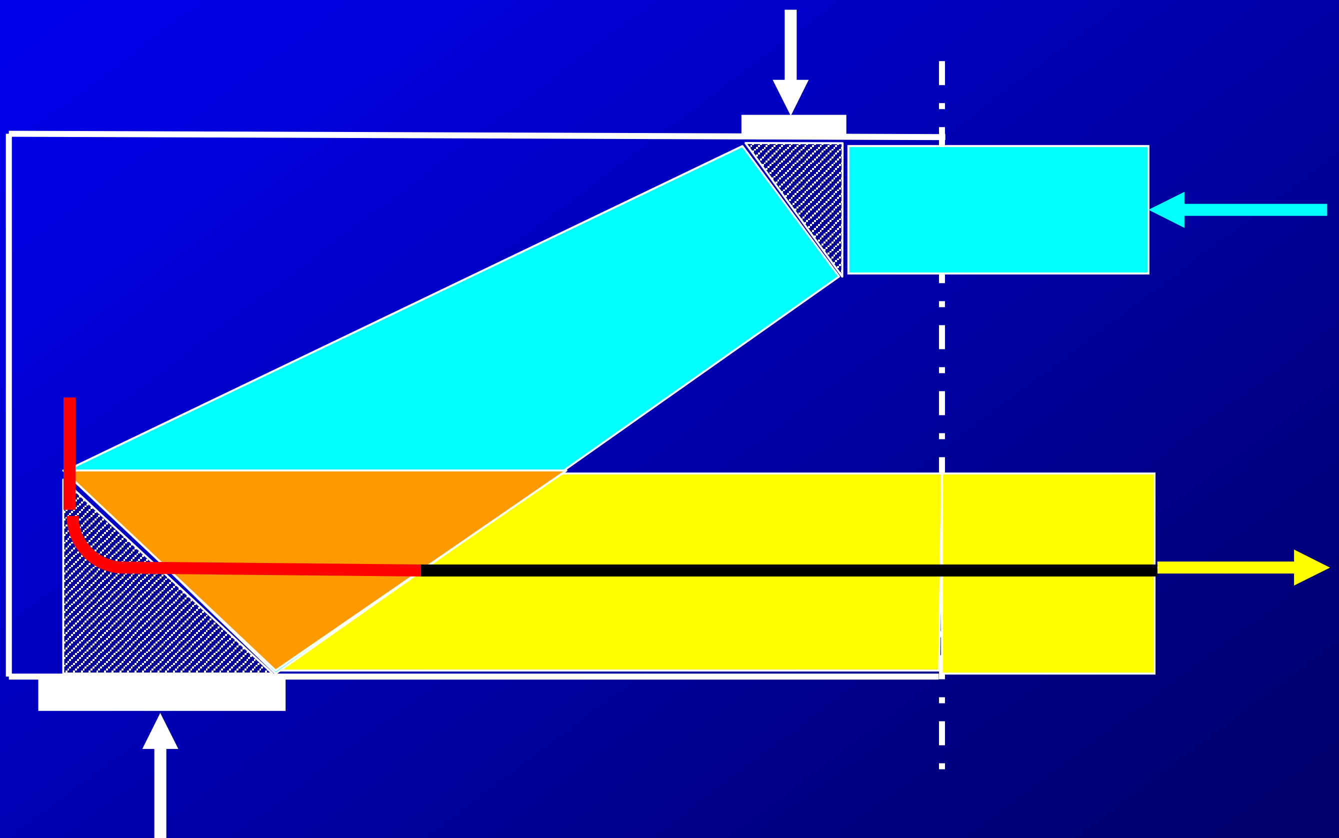


# Strength of Ties

- Strength = $\phi A_s f_y$, where $\phi = 0.75$*
- Anchorage of ties at nodes is a major concern

**A constant value of $\phi = 0.75$ is to be used for sizing the strut-and-tie model, but the use of $\phi = 0.75$ to also select the reinforcement may need further examination within the appropriate Code subcommittee.*

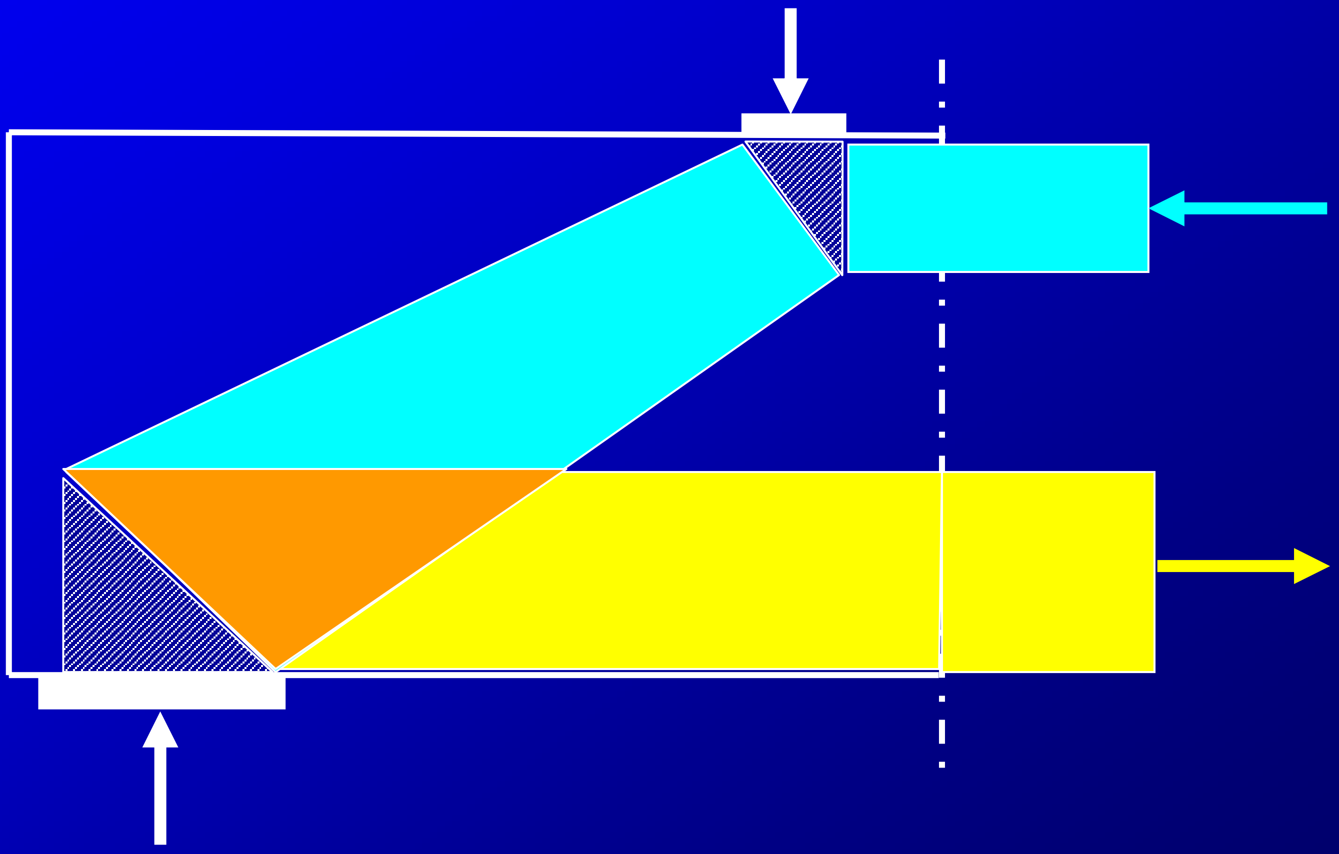
# Anchorage Check



# Nodal Zone Shape and Dimensions

- Width of compression face is same as width of strut connecting to nodal zone (smaller allowable strength governs)
- Height (width of face perpendicular to tie force) of nodal zone is equal to tie force divided by effective compressive strength of the concrete in the node

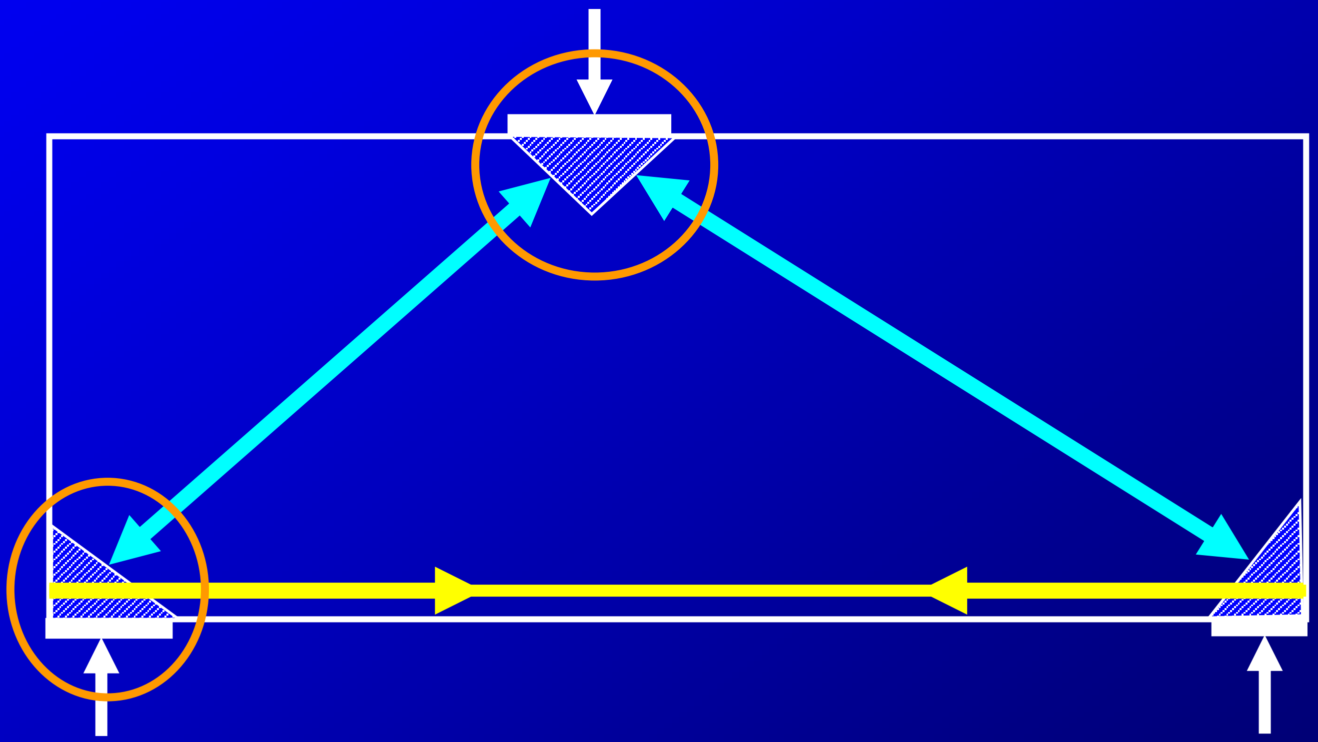
# Node Shape and Size



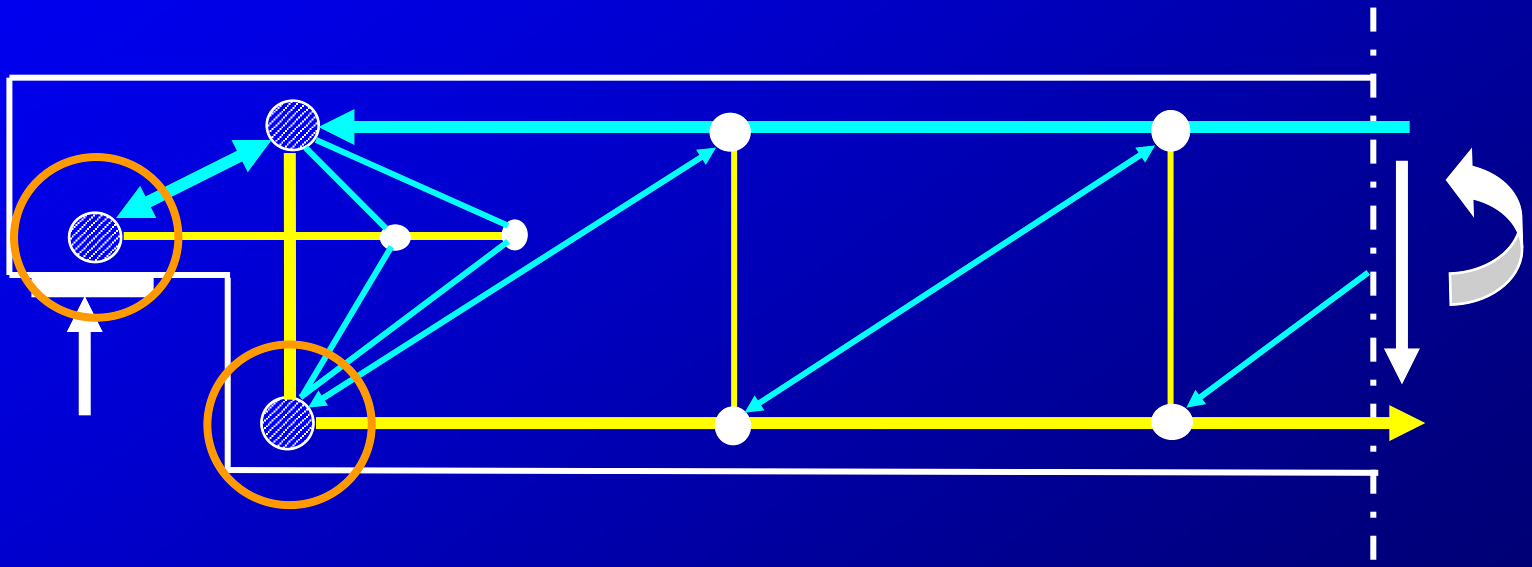
# Effective Strength of Nodal Zones

- Function of type of members connected to the node
- Possible combinations are CCC, CCT and CTT
- Strength can be enhanced by addition of confinement reinforcement

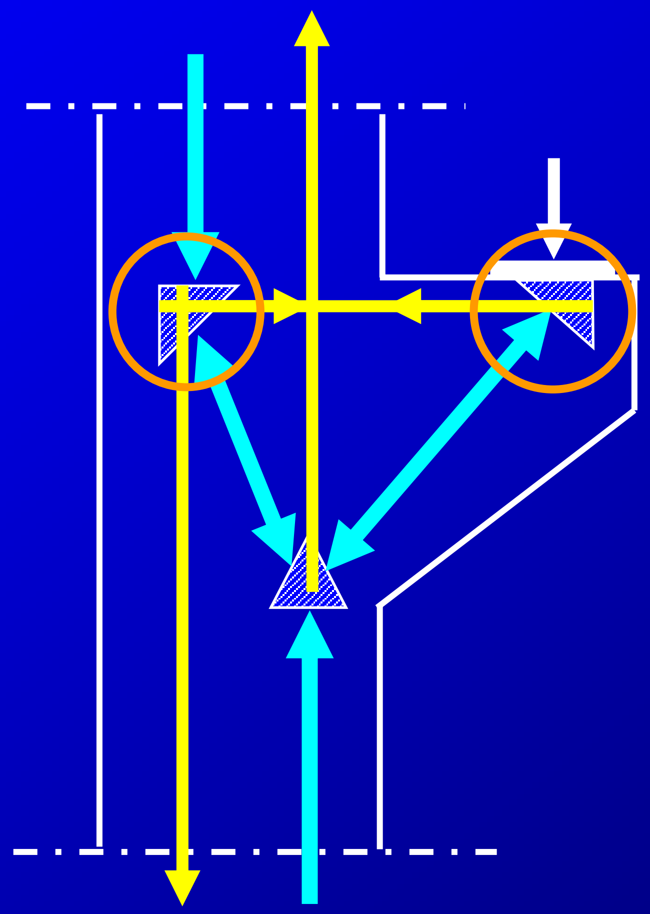
# Examples of CCC and CCT Nodes



# Examples of CCT and CTT Nodes



# Examples of CCT and CTT Nodes



# Strength of Nodes

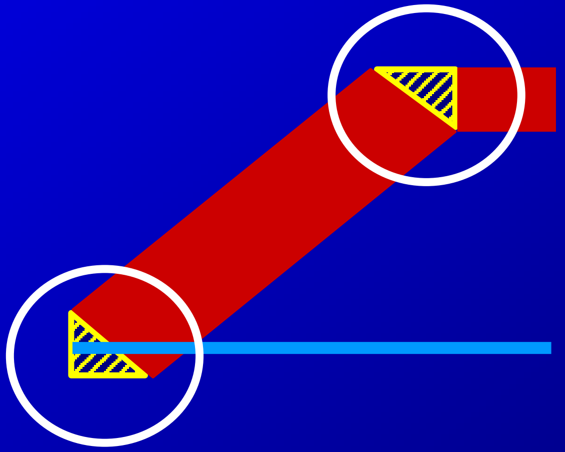
$$\phi F_{nn} \geq F_u$$

$$\phi = 0.75$$

$$F_{nn} = f_{cu} A_n$$

$$A_n = area\ of\ node\ face \perp to\ force\ F_u$$

$$f_{cu} = 0.85\, \beta_n\, f'_c$$



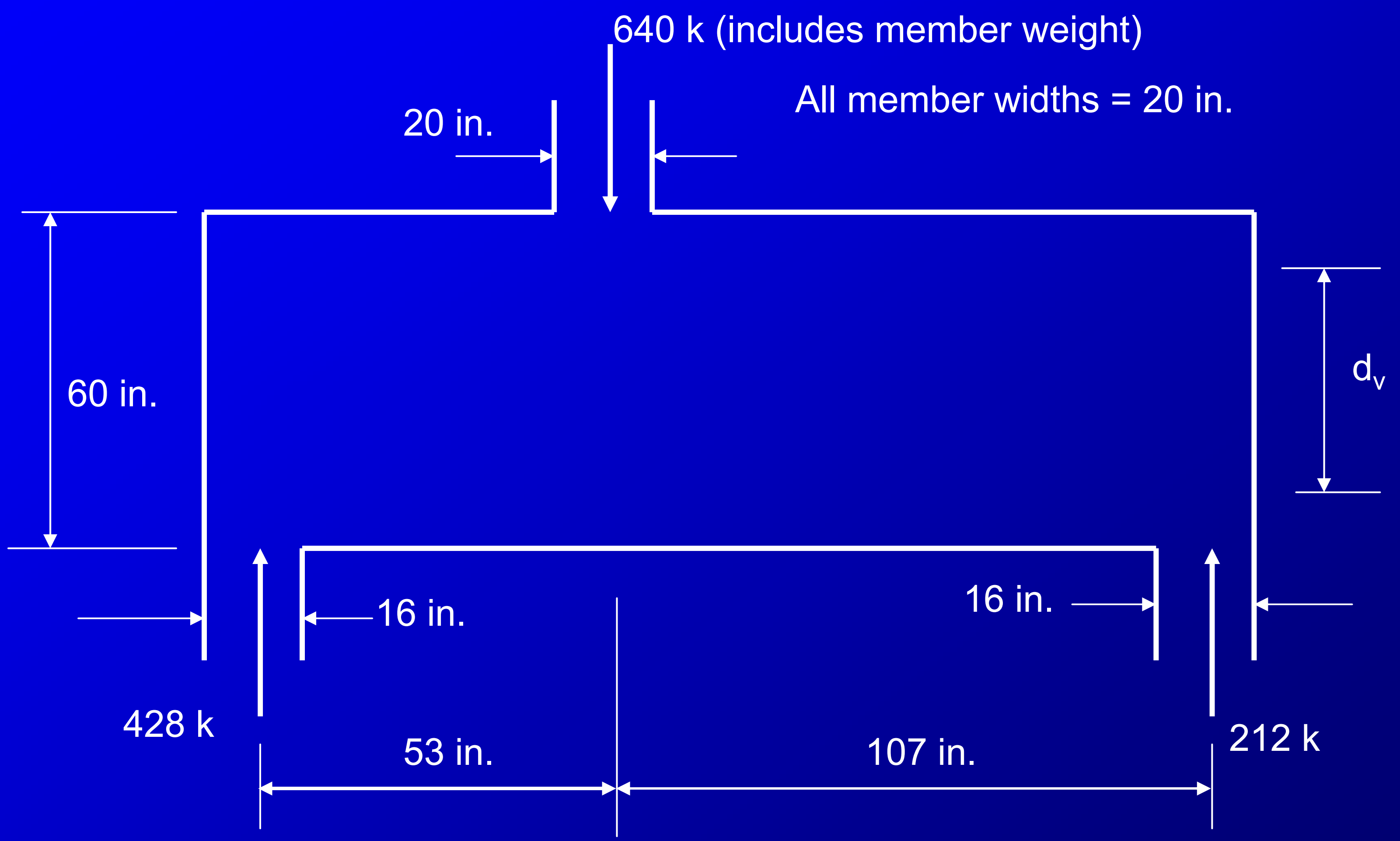
# Recommended $\beta_n$ Values (Nodes)

- 1.0 - CCC node
- 0.8 - CCT node
- 0.6 - CTT node

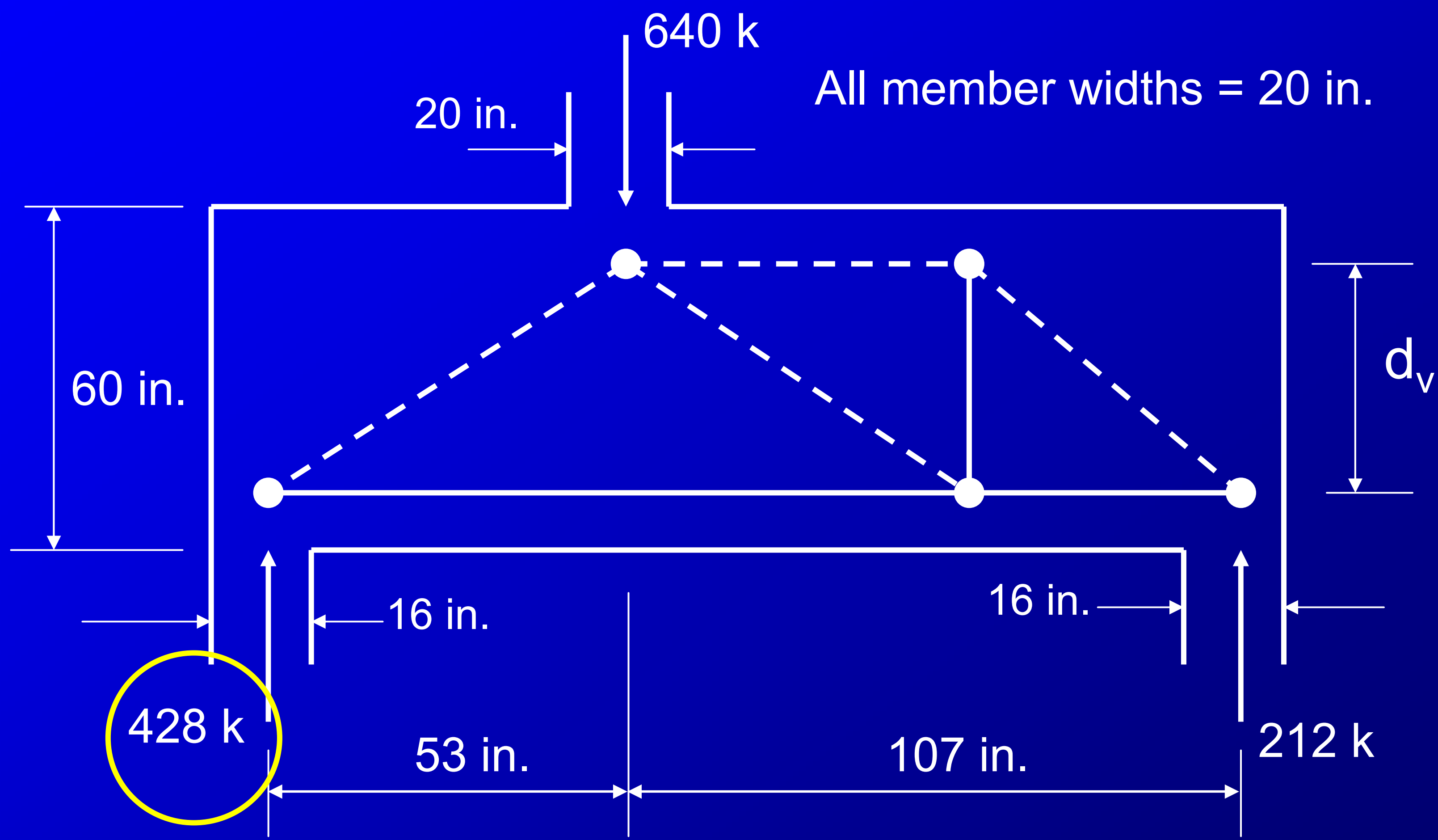
# Use of STM in the ACI Code

- Appendix A in 318-05 Code; Parts of the appendix may be moved into regular code chapters in a later edition of the Code.
- Currently replaces deep beam design procedure in prior editions of Code.
- Is listed as alternate procedure in several sections of the code.

# Example: CI Magazine, May 2003



# Beam Dimensions and Initial Truss Model



# Check Max. Allowable Shear Force

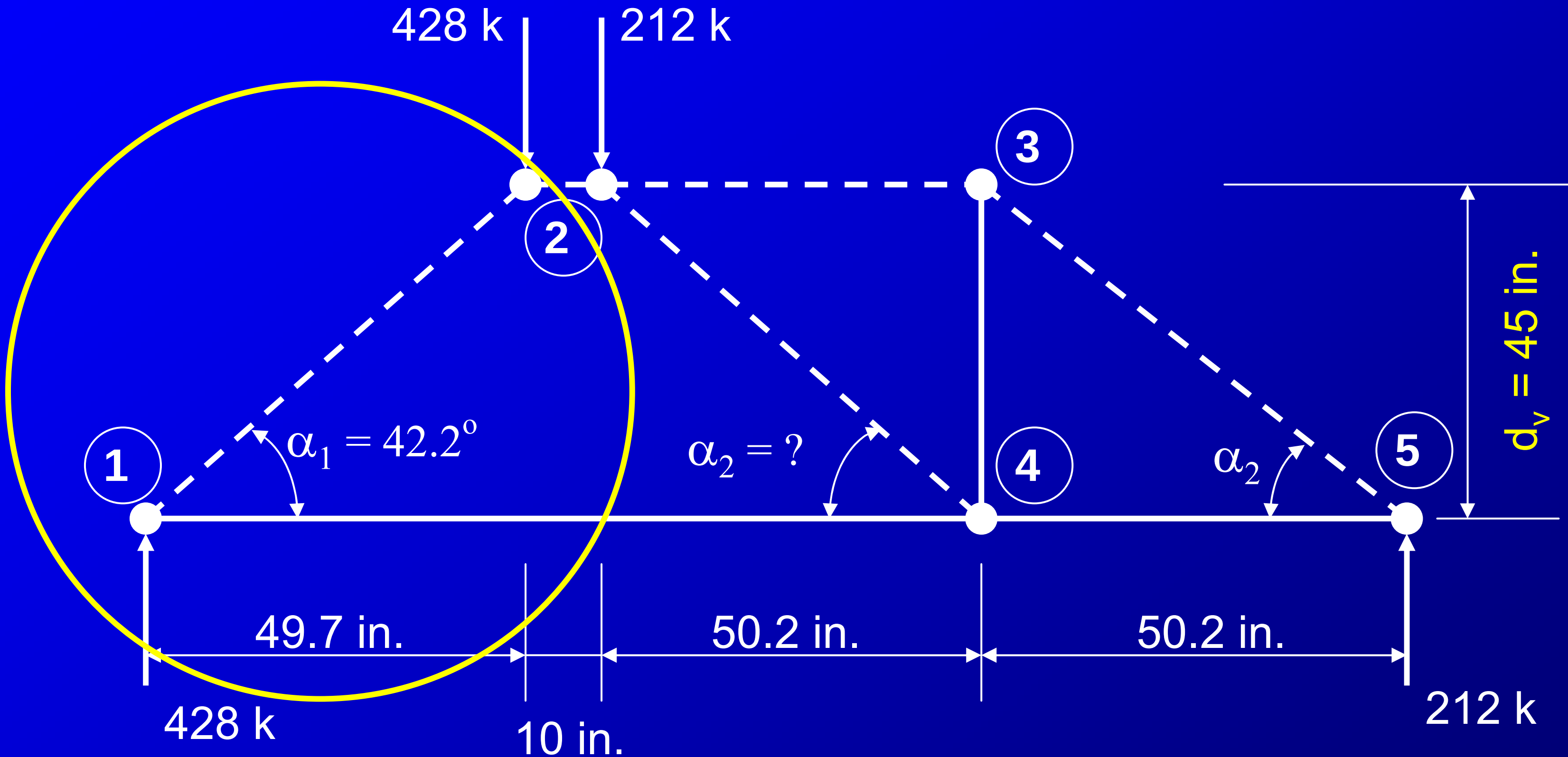
Max. shear force = 428 k

$$\text{Max. All. Shear Force} = \phi \cdot 10\sqrt{f_c'} \cdot b_w d$$

$$\phi \cdot V_n(\max) = 0.75 \times 10 \times \frac{\sqrt{4000}}{1000} \times 20 \times 54 = 512 \text{ k (o.k.)}$$

Assumed that d ≈ 0.9×h

# Initial Truss Geometry

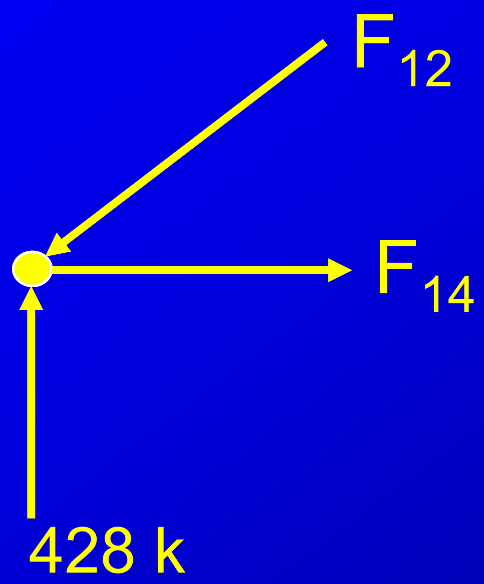


# Establish Truss Geometry: Start with left portion of the beam

Assume $d_v$ = 60 in. – 2(7.5 in.)
= 45 in. (1140 mm)
(Assumes node heights = 15 in.)

Thus $\tan \alpha_1$ = (45in./49.7in.),
implies that $\alpha_1$ = 42.2 deg.

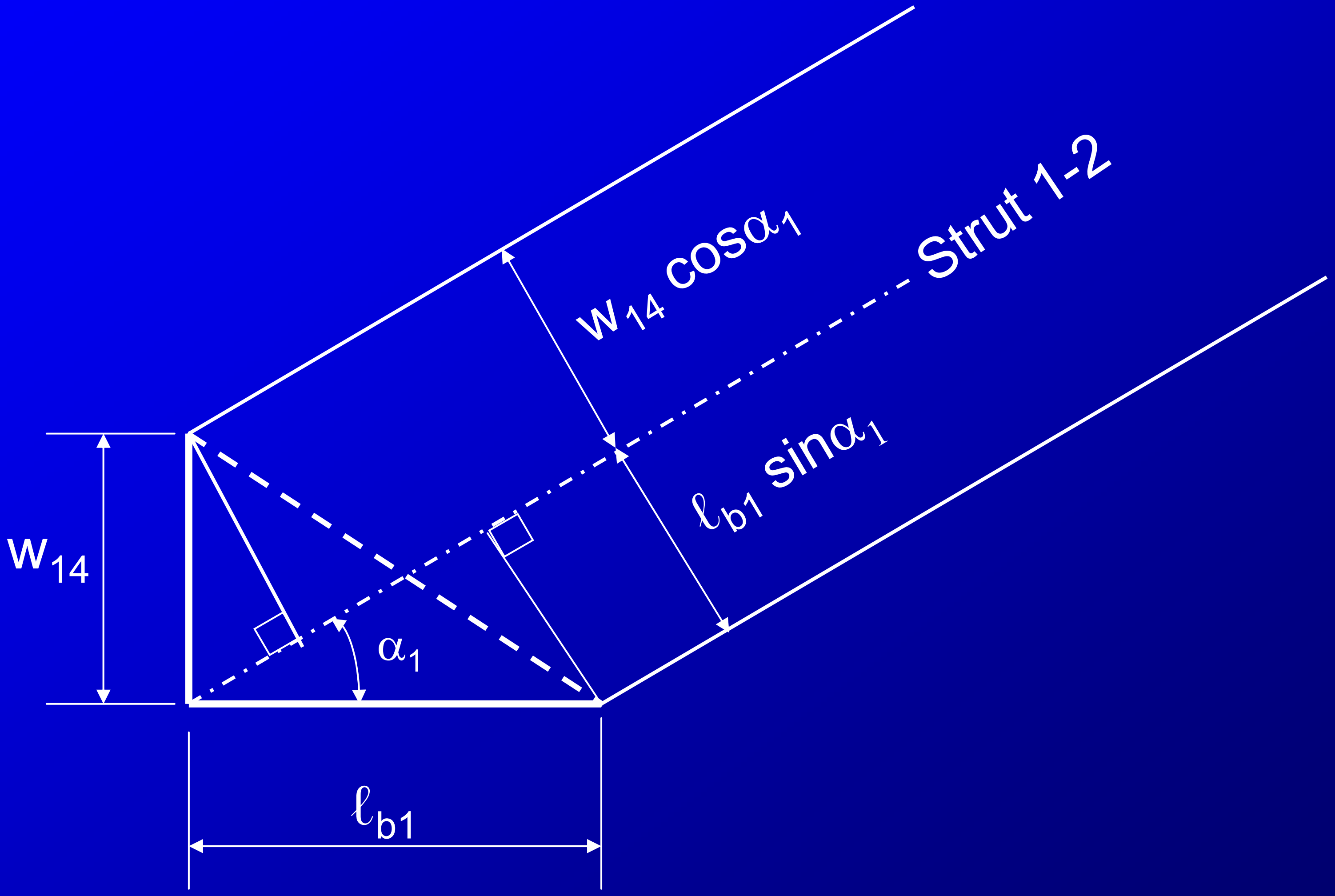
# Establish Equilibrium at Node 1




$$\Sigma F_y = 428 - F_{12} \times \sin \alpha_1 = 0$$
$$F_{12} = 637 \text{ k}$$

$$\Sigma F_x = F_{14} - F_{12} \times \cos \alpha_1 = 0$$
$$F_{14} = 472 \text{ k}$$

# Geometry and dimensions of Node 1 and Strut 1-2



# Establish/Check Dimensions at Node 1

$\phi\, f_{cu}(1) = \phi\,(0.85)\,\beta_n\, f'_c = 0.75 \times 0.85 \times 0.80 \times 4$

$\phi\, f_{cu}(1) = 0.75 \times 2.72 = 2.04$ ksi

$$f\,(base) = \frac{R_\ell}{b_w \cdot \ell_{b1}} = \frac{428}{20 \cdot 16} = 1.34 \text{ ksi (o.k.)}$$

$$w_{14} = \frac{F_{14}}{b_w \cdot \phi \cdot f_{cu}(1)} = \frac{472}{20 \times 0.75 \times 2.72} = 11.6 \cong 12 \text{ in.}$$

# Establish/Check Width and Strength of Strut 1-2

$$w_{12} = w_{14} \cdot \cos\left(\alpha_1\right) + \ell_{b1} \cdot \sin\left(\alpha_1\right)$$

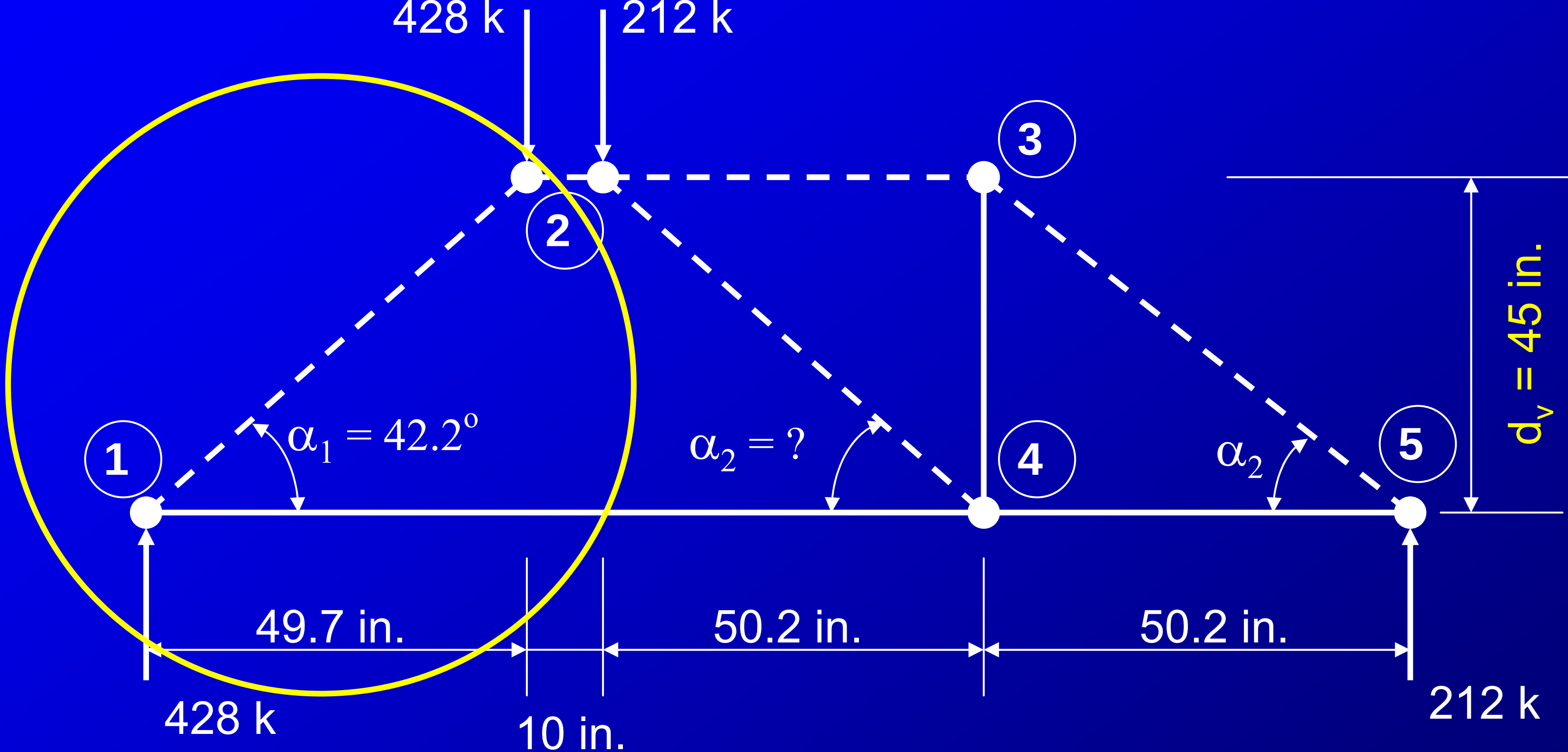
$$w_{12} = 8.89 + 10.8 = 19.7 \text{ in.}$$

$$f_{cu}(1\text{-}2) = 0.85 \times \beta_s \times f_c' = 0.85 \times 0.75 \times 4 = 2.55 \text{ ksi}$$

$$\phi F_{ns}(1\text{-}2) = \phi f_{cu}(1\text{-}2) \cdot w_{12} \cdot b_w$$

$$\phi F_{ns}(1\text{-}2) = 0.75 \times 2.55 \times 19.6 \times 20 = 757 \text{ k} \ \geq\ 637 \text{ k (o.k.)}$$

# Initial Truss Geometry



# Modified Truss Geometry and Member Forces

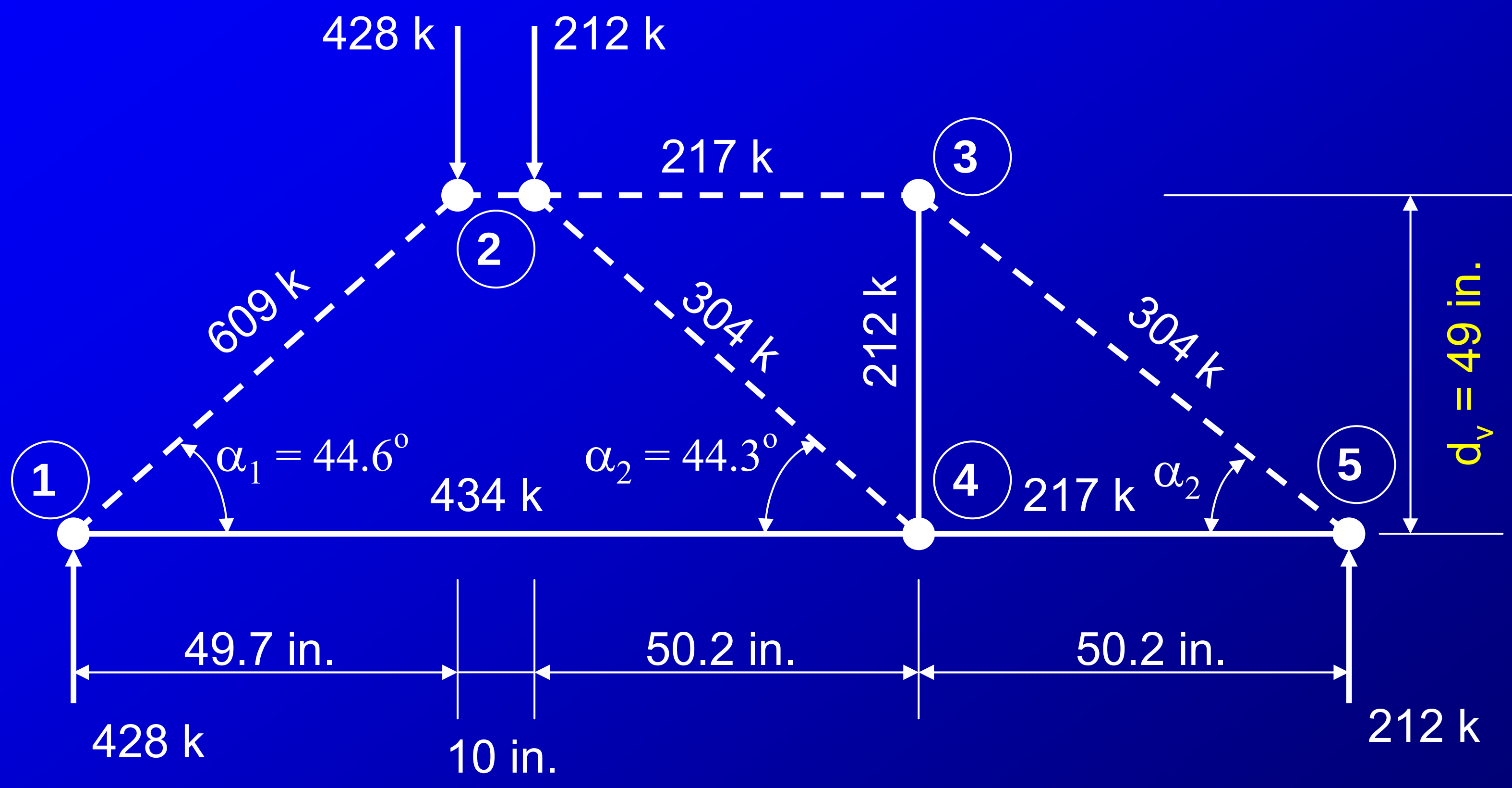
Thus, height of Node 1 = 12 in.

Because Node 2 is a CCC node, assume is has a total height of 10 in.

Then, $d_v = 60 – (12 + 10)/2 = 49$ in.

Reestablish truss geometry and member forces!

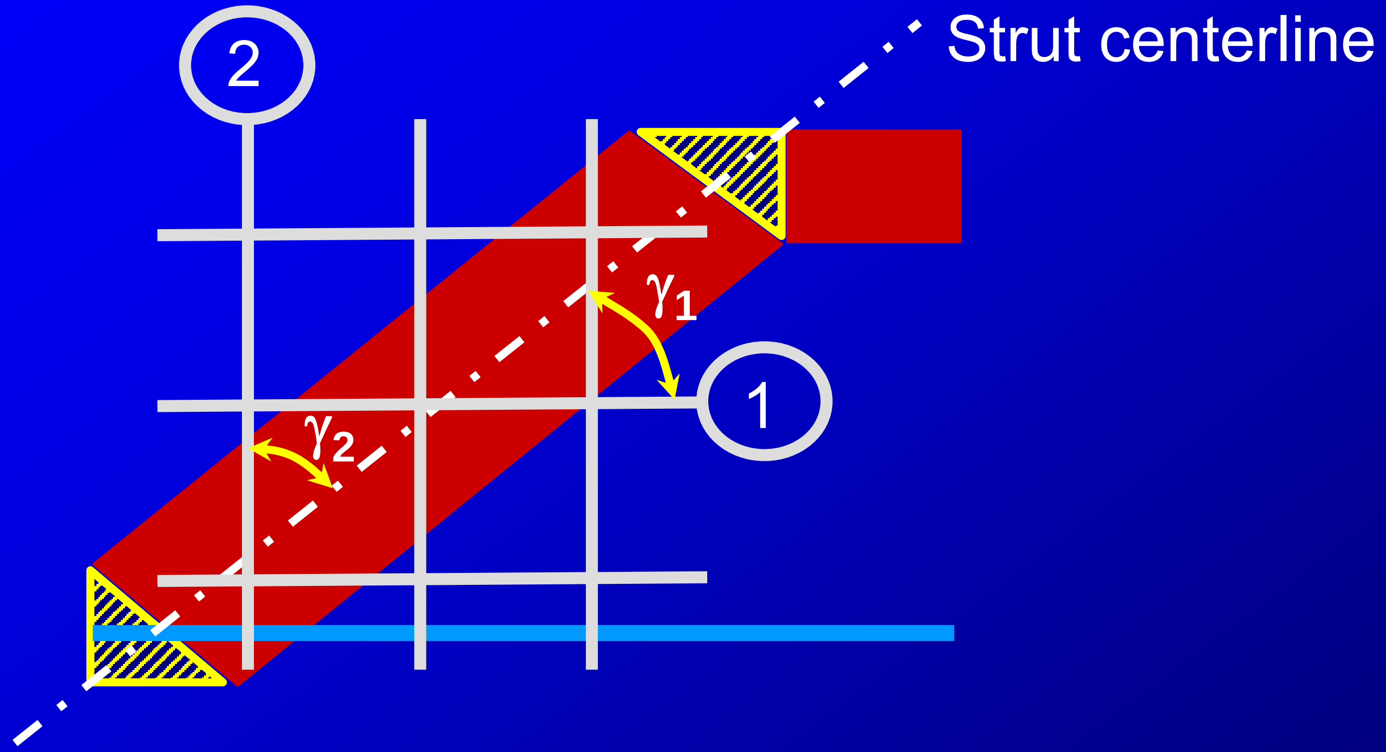
# Final truss geometry and member forces



# Minimum Reinforcement Crossing Strut 1-2 for $\beta_s = 0.75$

- This reinforcement is to control the growth and width of cracks crossing the strut.
- Because this is a deep beam, I recommend that you also satisfy Code Sections 11.8.4 and 11.8.5.
- Because this is a deep flexural member, the minimum skin reinforcement requirements of ACI Section 10.6.7 must be satisfied.

# Minimum Reinforcement Grid ($f'_c \leq 6000$ psi)




$$\Sigma \frac{A_{si}}{b\, s_i} \sin \gamma_i \geq 0.003$$

# Minimum Reinf. Crossing Strut 1-2

Vertical Reinforcement (stirrups):

Angle to strut axis, $\gamma_1 = 90° – 44.6° = 45.4°$

Using No. 4 stirrups with four legs

at s = 10 in. (< 12 in. and < d/5)

$$\rho_v = \frac{A_v}{b_w \cdot s} = \frac{4 \times 0.20}{20 \times 10} = 0.0040 \geq 0.0025$$

Thus, $\rho_v$ ($\sin \gamma_1$) = 0.00285

# Minimum Reinf. Crossing Strut 1-2

Horizontal Reinforcement (layers):

Angle to strut axis, $\gamma_2 = 44.6°$

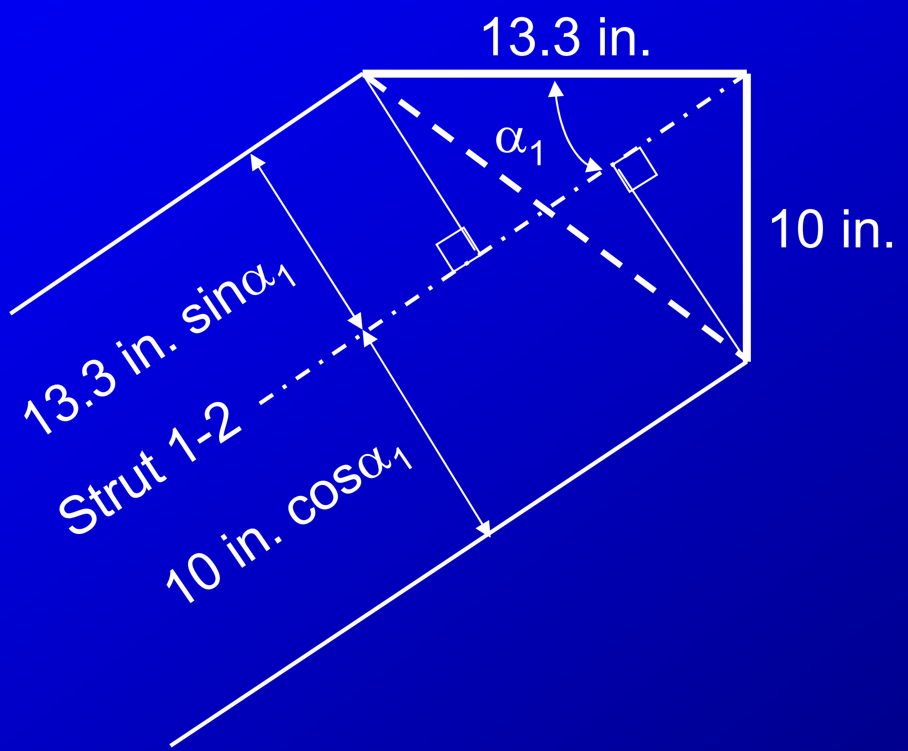
Using two No. 4 bars per layer with a spacing of 8 in. between layers to satisfy 10.6.7 and 11.8.5

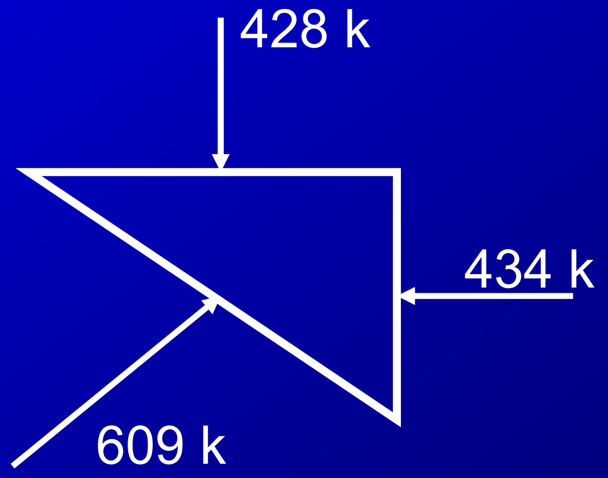
$$\rho_h = \frac{A_h}{b_w \cdot s} = \frac{2 \times 0.20}{20 \times 8} = 0.0025 \geq 0.0015$$

Thus, $\rho_h (\sin \gamma_2) = 0.00176$

And $\rho_v(\sin\gamma_1) + \rho_h(\sin\gamma_2) = 0.00461 > 0.003$

# Geometry, forces and dimensions for left part of Node 2



# Force Checks on each face of Node 2

Permissible stress in Node 2:

$$f_{cu}(2) = 0.85\beta_n f_c' = 0.85 \times 1.0 \times 4 = 3.40 \text{ ksi}$$

Check top of node:

$$\phi f_{cu}(2) \cdot b_w \cdot 13.3 = 678 \text{ k} \;\geq\; 428 \text{ k (o.k.)}$$

Check internal vertical face:

$$\phi f_{cu}(2) \cdot b_w \cdot 10 = 510 \text{ k} \;\geq\; 434 \text{ k (o.k.)}$$

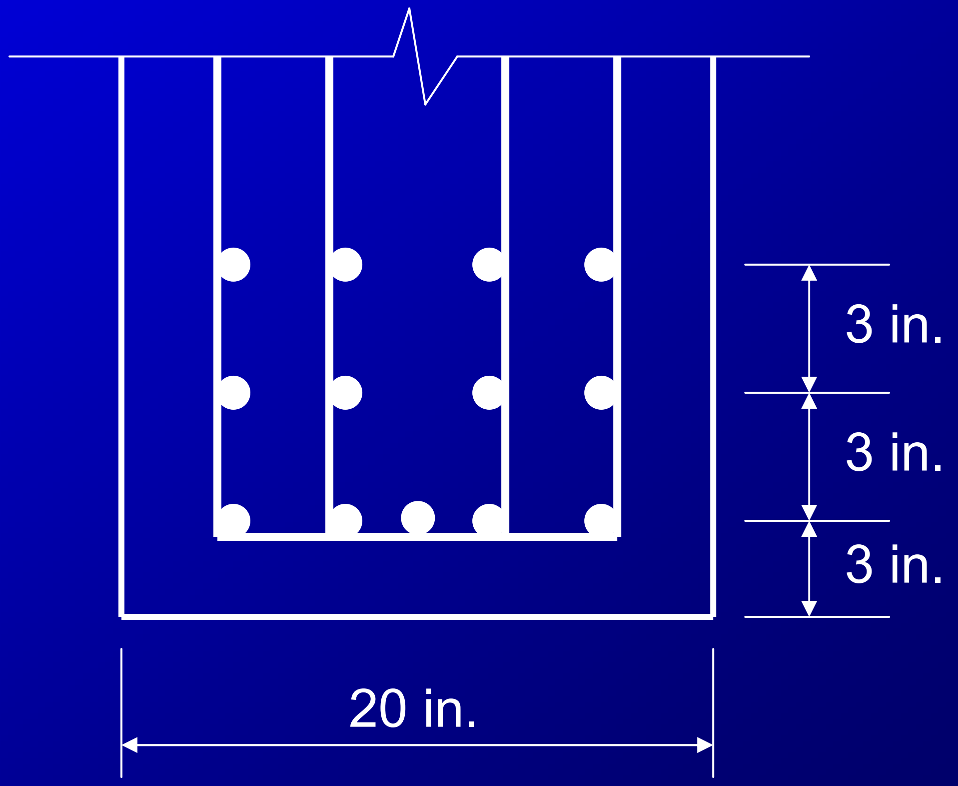
Check top of strut 1-2:

$$\phi f_{cu}(1\text{-}2) \cdot w_{12} \cdot b_w = 631 \text{ k} \;\geq\; 609 \text{ k (o.k.)}$$

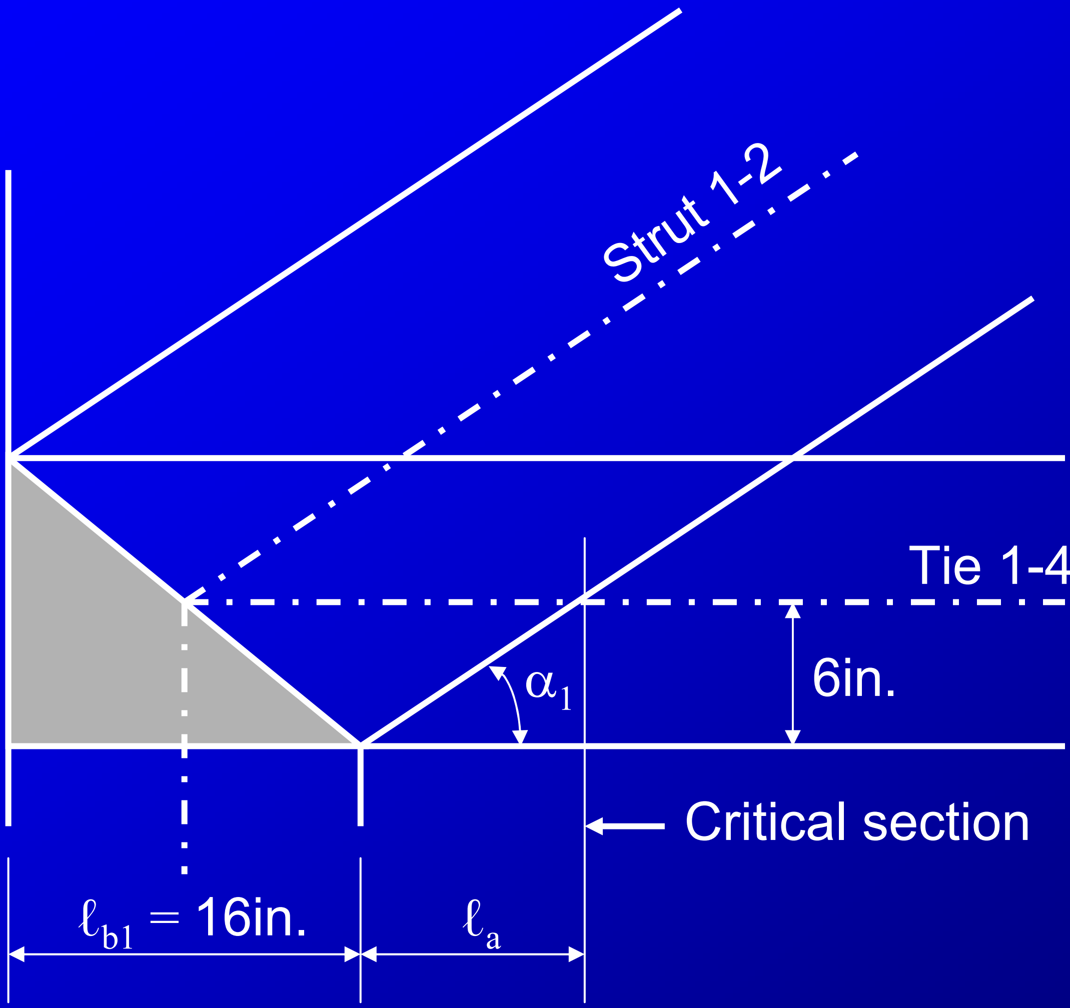
# Select Reinforcement for Tie 1-4

$$A_s(req'd.) = \frac{F_{14}}{\phi \cdot f_y} = \frac{434}{0.75 \times 60} = 9.64 \text{ in.}^2$$

Select 13 No. 8 bars,
$A_s$ = 10.3 in.$^2$



# Check Anchorage at Node 1




$\ell_a = 6$ in./tan $\alpha_1$
$\ell_a = 6.09$ in.

Total anchorage
Length $\approx$ 22.1 in.

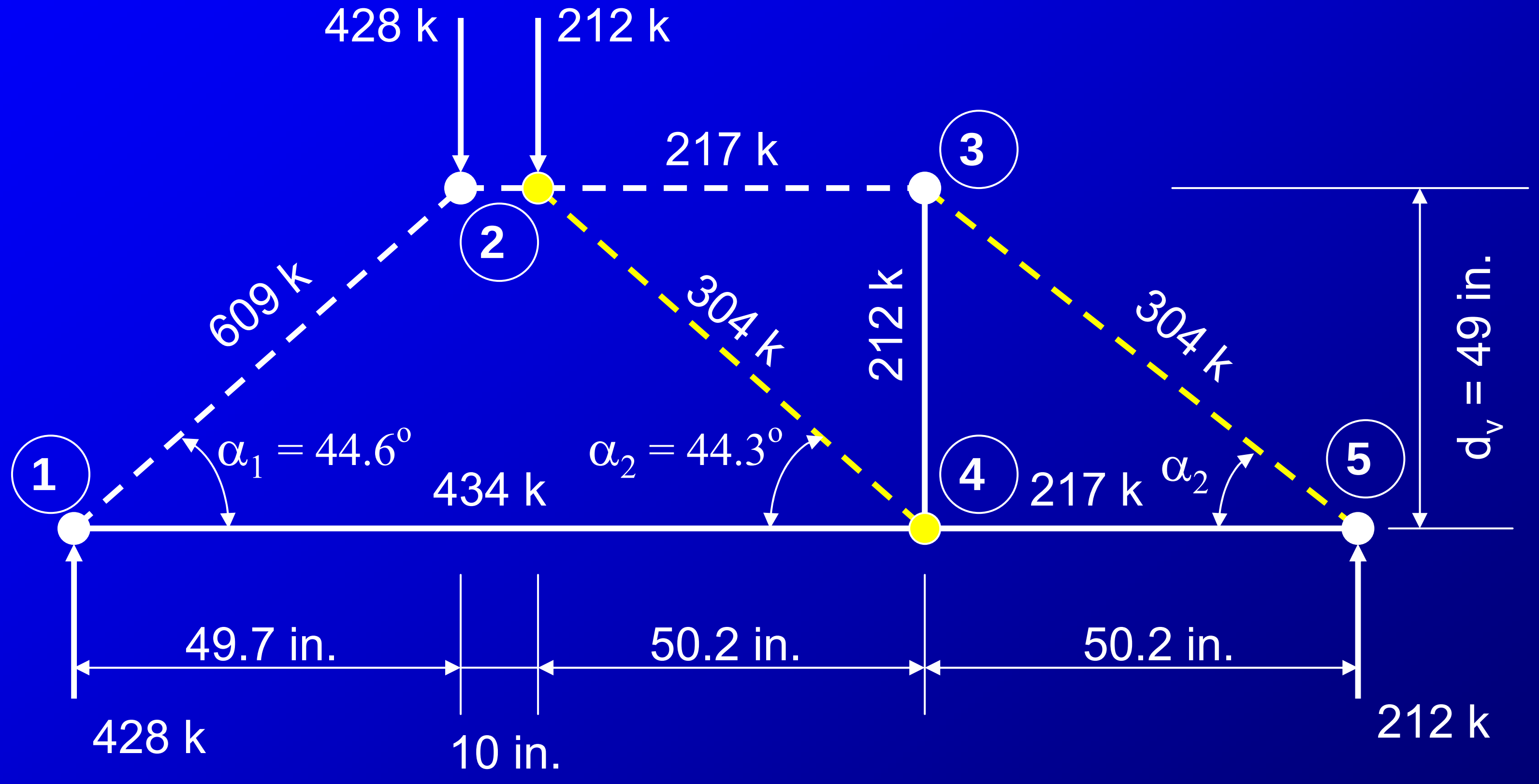
# Check Anchorage at Node 1

$$\ell_{dh} = \frac{0.02\psi_e \lambda f_y}{\sqrt{f_c'}} d_b = \frac{0.02 \times 1 \times 1 \times 60,000}{\sqrt{4000}} \times 1.0$$

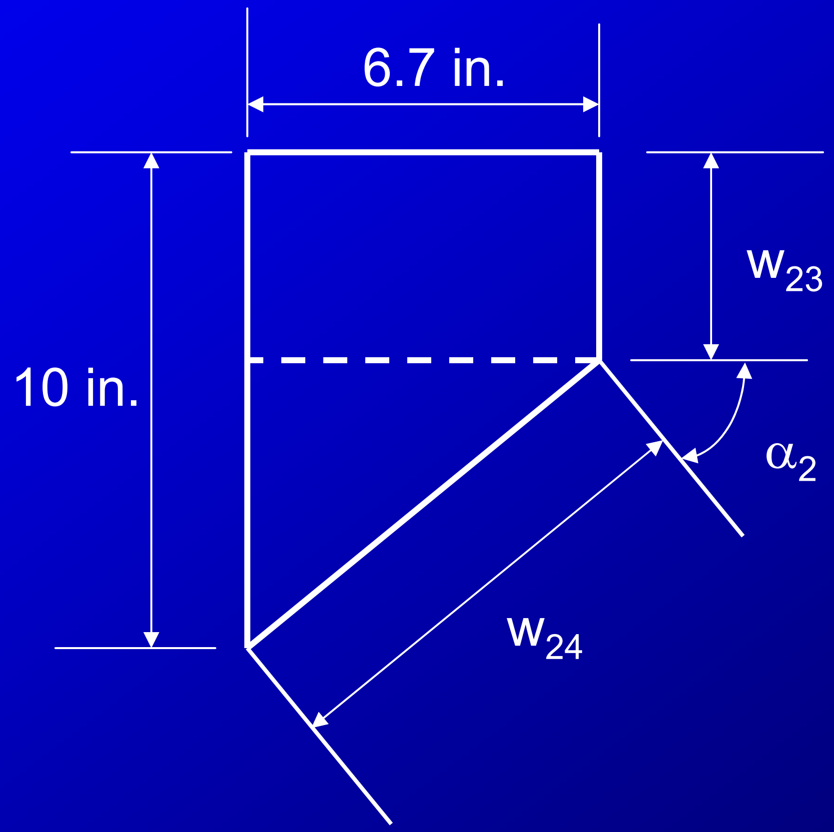
$$\ell_{dh} = 19.0 \text{ in. } (> 8\text{d}_\text{b} \text{ and} > 6 \text{ in.})$$

Although $\ell_{dh}$ is less than 22 in., this would be a tight fit if only 90° hooks were used. In-plane 180° hooks could be used for some bars to partially relieve this rebar detailing problem. The use of mechanical anchorage devices could also be considered.

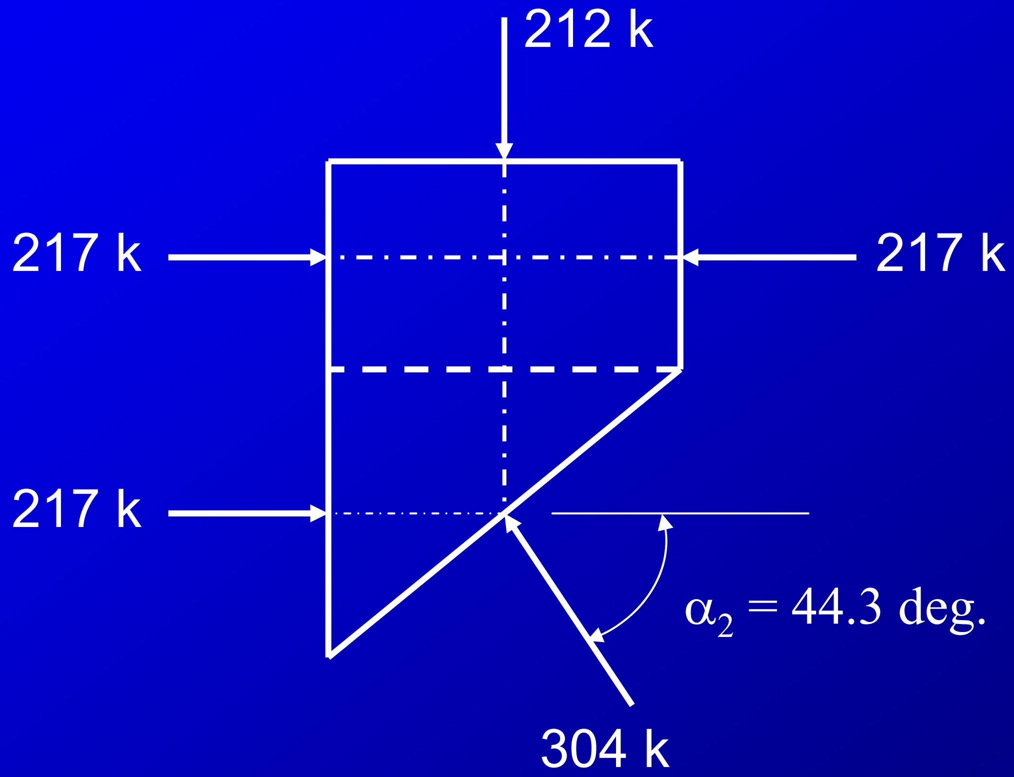
# Comments on right half of truss



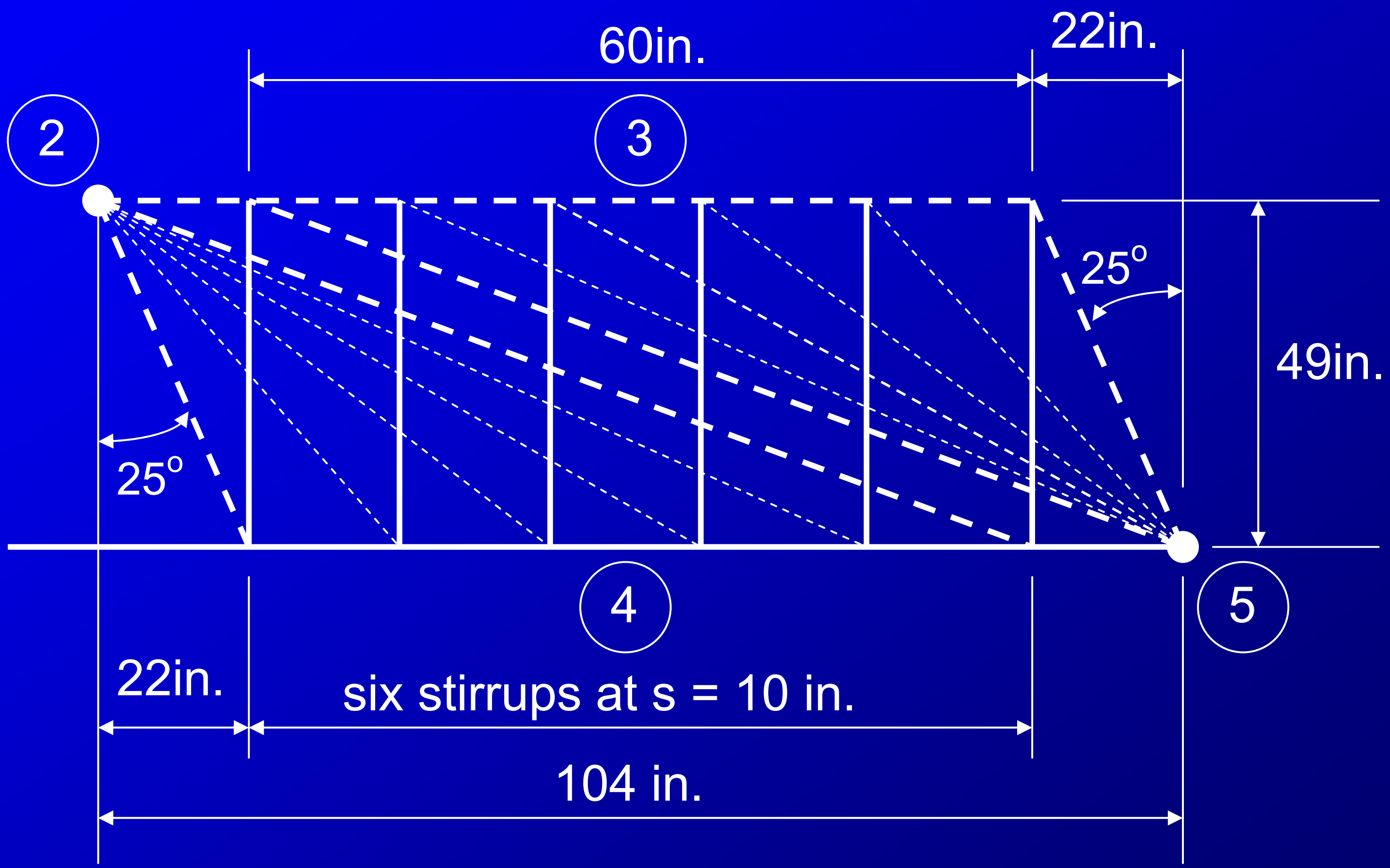
# Geometry for right half of Node 2



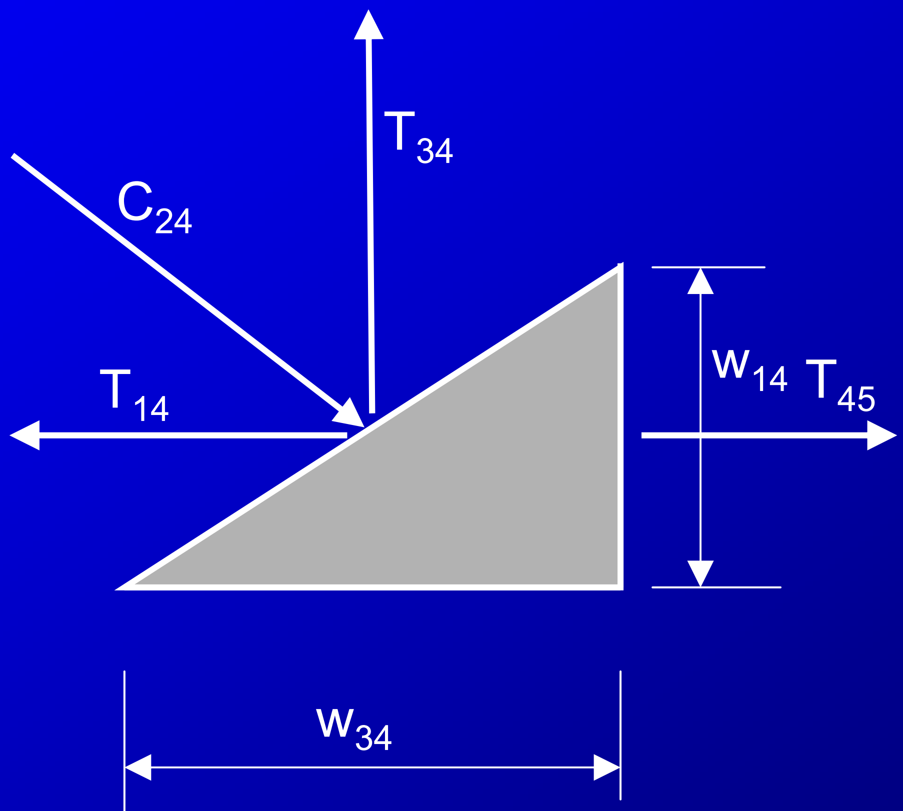
# Vector equilibrium for right half of Node 2



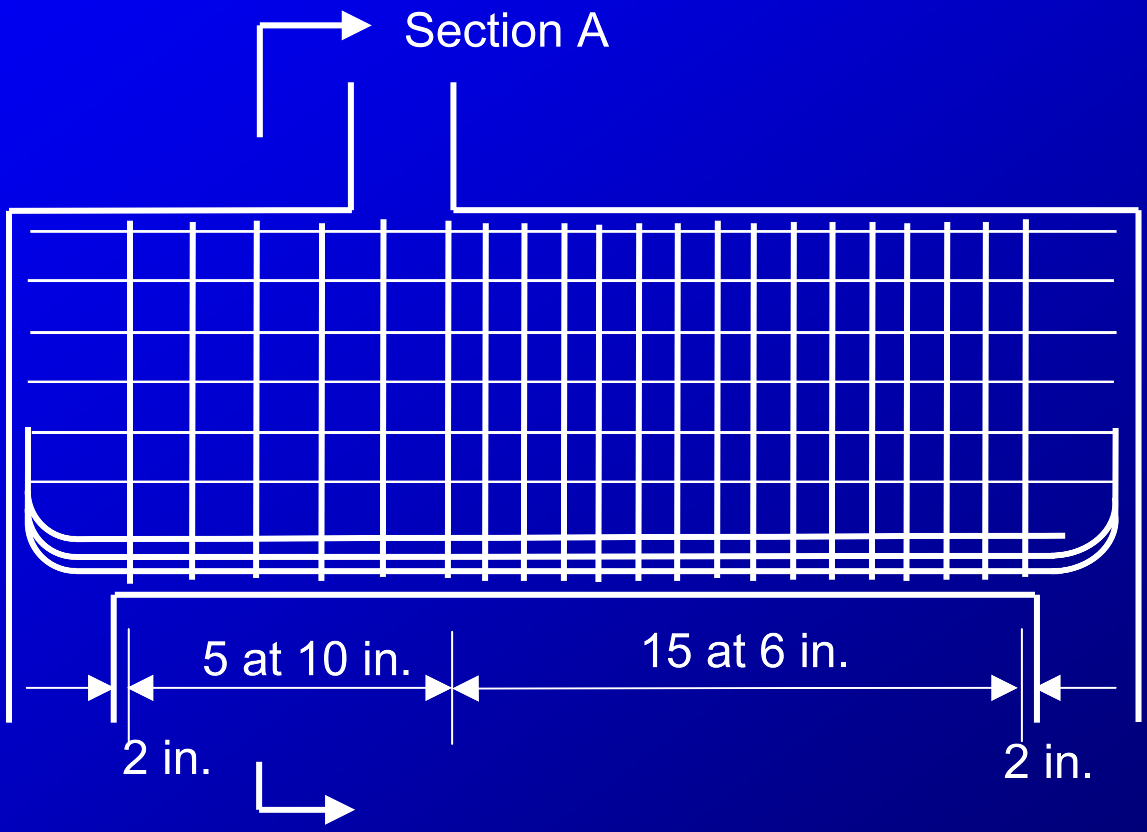
# Analysis of fan-shaped struts 2-4 and 3-5, and tie 3-4



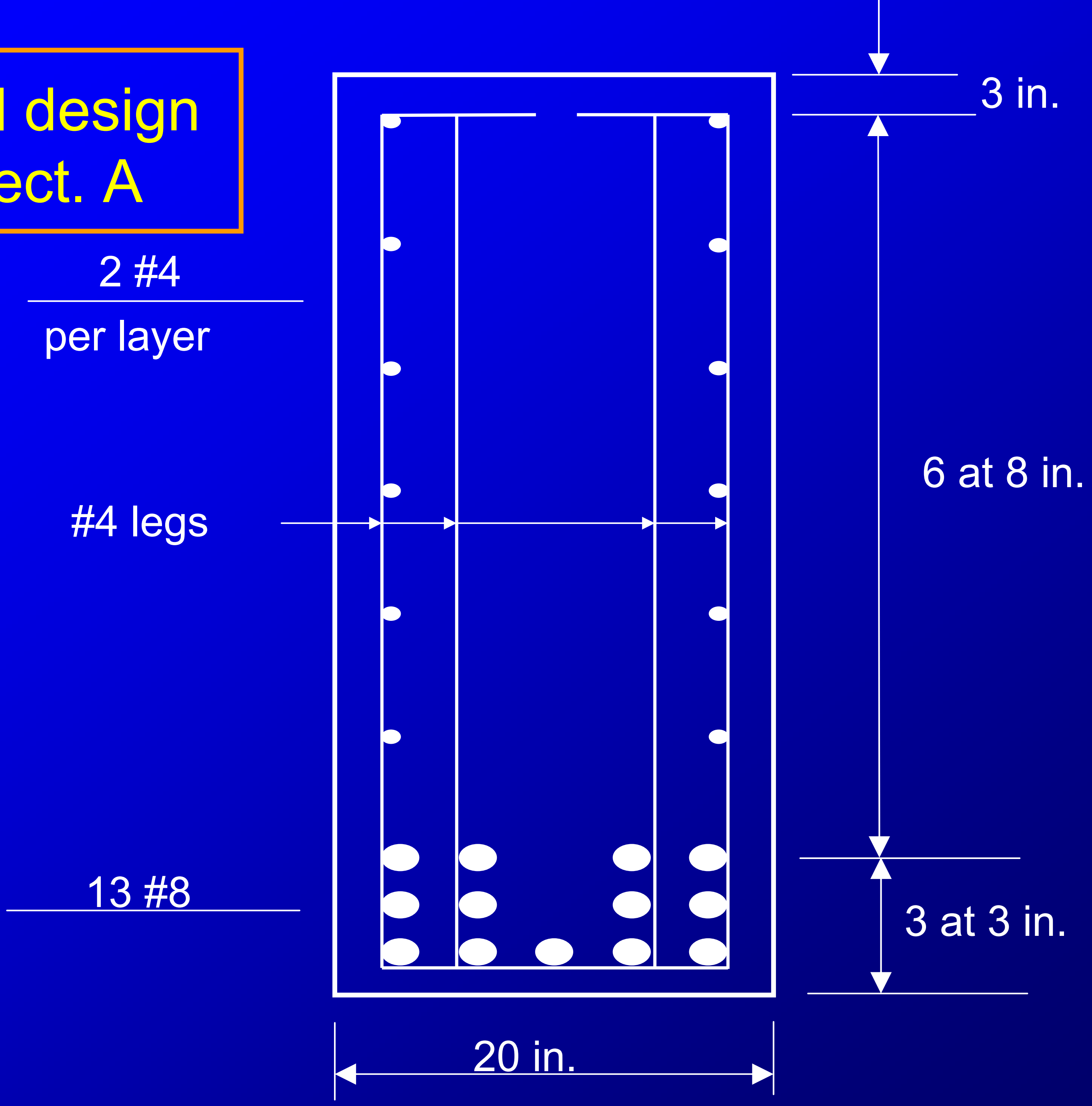
# Analysis of Node 4



# Final design of longitudinal and transverse steel



Final design
At Sect. A



Thank You

Gracias

Questions ?

Preguntas ?